The Strict Proof that General Relativity can not Describe the Ellipse Orbits of Planets in the Solar System

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Abstract In general relativity, the arc length s is used as a variation parameter to establish geodesic equations. In the weak gravitational field of the solar system, the approximate condition $ds = cdt$ is used to deduce the motion equations of planets. It is pointed out in this paper, that this condition is wrong. It will causes serious contradiction by the self-entanglement of theory. In order to avoid the uncertainty of approximate calculation, the arbitrary variation parameter is used to establish the geodesic equations. In this way, it is proved strictly that the motion equations of general relativity can only describe the parabola orbit, can not describe the ellipse and hyperbole orbits. So general relativity is invalid for the planet's motions in the solar system. Bu means of the numerical method of computer, it is proved further that besides the perihelion precession of Mercury, general relativity would cause serious deformations of planet's orbits. The distance of perihelion would increase 94000Km and the distance of aphelion would decrease 74000Km for the Mercury moving a circle around the sun. The change of semi-focal chord was about several kilometers for the Mercury moving a circle around the sun. The change was asymmetric and cumulative so that the orbit of Mercury would collapse. However, no such deformations are observed. The perihelion precession of Mercury is discussed at last. It is pointed out that because the calculation error for other planet's perturbations on the Mercury's orbit is far greater than 43'' a centaury, the calculation of general relativity on perihelion precession of Mercury is meaningless. Three direct examples that general relativity can not explain the precessions of double stars are provided in the paper. The conclusion is that the revision of general relativity on the Newtonian theory of gravity is impossible. The Einstein's gravity theory of curved space-time is untenable. The gravity theory of modern physics should be reestablished based on the flat space-time through the revision of the Newtonian theory of gravity.

Key words： General Relativity, the Newtonian theory of gravity, Riemann Geometry, Geodesic Equation, Schwarzschild metric, Ellipse Orbit, Parabola Orbit, Perihelion Precession of Mercury.

1. Introduction

In general relativity, the arc length s is used as a variation parameter to establish geodesic equations. Based on the Schwarzschild Solution in the gravitational field with spherical symmetry, the planet's motions equation is obtained in the solar system. By using polar coordinate system and let $u = 1/r$, the motion equation of planets in general relativity is 【1】,【2】

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$$\left(\frac{du}{d\varphi}\right)^2 = -\frac{\alpha}{2c^2bh^2} + \frac{\alpha}{c^2h^2}u - u^2 + \alpha u^3 = f(u) \qquad (1)$$

Her M is the solar mass and the constant $\alpha = 2GM/c^2 = 2.95\times10^3$ is the solar gravitational radius. $h^2 = GMb(1-e^2)/c^2$ and b is the semi-focal chord of planet's ellipse orbit. Comparing with the Newtonian eqaution of gravity, there is a revised item αu^3 in (1). By using (1) to calculate the perihelion precession of Mercury, the result was considered to agree with the observations. So, the revision of general relativity was thought to be correct. This result is considered as the most important successful of general relativity.

However, the approximate condition $ds = cdt$ was used in the deduction process of (1) which causes following serious problems:

1. The approximate condition $ds = cdt$ made the Schwarzschild metric unequal on the two sides of equal sign and causes serious contradiction by the self-entanglement of theory. So that the equation of geodesic equations can not be used and the motion equation of general relativity can not be obtained.

2. In order to resolve these problems, arbitrary variation parameter is used to establish geodesic equations and the motion equation of planets in the solar system is deduced. It is proved strictly that the planet's orbits can only be parabola, rather than ellipse and hyperbole. So the motion equations of general relativity are invalid in the solar system. This result is fatal and incurable for the Einstein's gravity theory of curved space-time.

The method of numerical calculation is used to study the equation of planet motion of general relativity. It is disclosed that besides the perihelion precession, general relativity would cause the serious deformation of planet's orbit. The distance of perihelion would increase 94000Km and the distance of aphelion would decrease 74000Km for the Mercury moving a circle around the sun. The change of semi-focal chord is about several kilometers for the mercury moving a circle around the sun. The change was asymmetric and cumulative so that the orbit of Mercury would crash. Comparing with theses deformations, the perihelion precession of Mercury becomes completely insignificant. However, no such deformations are observed.

At last, the perihelion precession of Mercury is discussed. Due to following four reasons, the calculation of general relativity is meaningless.

1. General relativity can not describe the ellipse orbits of planets.

2. The rough method was used to calculate the perturbations of other planets on the Mercury's motion. The error caused by the method may be far greater than the calculation value of 43" a century by general relativity.

3. If the effect of special relativity is considered in the Newtonian theory of gravity, the precession value of 43" a century can be deduced. If effect of special relativity is considered in the precession and nutation of the earth and other planet's perturbation, more revisions will be caused. All of these have not been considered in general relativity.

4. There are three direct proofs to show that general relativity can not explain the precessions of double stars.

The conclusion of this paper is that The Einstein's gravity theory of curved space-time is untenable. The gravity theory of modern physics should be reestablished based on the flat space-time through the revision of the Newtonian theory of gravity.

2. The Newtonian theory in spherical symmetry gravity field

In the gravitational field with spherical symmetry, the Newtonian gravity is

$$\vec{F} = -\frac{GMm\vec{r}}{r^3} \qquad\qquad F = -\frac{GMm}{r^2} \tag{2}$$

By using polar coordinate system, the motion equation of gravity is

$$m(\ddot{r} - r\dot{\varphi}^2) = F \tag{3}$$

$$m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}) = \frac{m}{r}\frac{d}{dt}(r^2\dot{\varphi}) = 0 \tag{4}$$

From (4), we get $r^2\dot{\varphi} = L =$ constant. Here L represents the angle momentum of unit mass. Substituting it in (3), we get 【3】

$$\frac{d^2r}{dt^2} - \frac{L^2}{r^3} = -\frac{GM}{r^2} \tag{5}$$

Let $u = 1/r$, (5) can be transformed as following formula [3]

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{L^2} \tag{6}$$

(6) describes the orbit of planet, which is unrelated to time. The solution of (6) is

$$u = \frac{GM}{L^2}(1 + e\cos\varphi) \qquad \text{or} \qquad r = \frac{L^2/GM}{1 + e\cos\varphi} \tag{7}$$

Here e is the eccentricity ratio of orbit. The values $e < 0$, $e = 0$ and $e > 0$ represent the orbits of ellipse, parabola and hyperbole individually. The Kepler kinetic energy integral for the ellipse orbit of planet is

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = -\frac{GM}{b} + \frac{2GM}{r} \tag{8}$$

It is actually the energy conservation formula of planet’s ellipse motion. For the general motion of planet in the solar system, the formula of energy conservation is

$$\frac{1}{2}m(\dot{r}^2 + r\dot{\varphi}^2) - \frac{GMm}{r} = E_0 \tag{9}$$

Here E_0 is the total mechanical energy. For the orbit of ellipse, $E_0 = -GMm/2b$ in which b is the major semi-axis. For the orbit of parabola, $E_0 = 0$. For the orbit of hyperbole, $E_0 = +GMm/2a$ in which a is the distance between the perihelion and the center point of coordinate system.

3. The proof that General relativity can not describe ellipse orbits

3. 1 The geodesic equation of general relativity

Einstein thought that space-time was curved when material existed and used following equation to describe gravity field

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} \tag{10}$$

Here $R_{\mu\nu} = R^{\sigma}_{\mu\nu\sigma}$ is Ricci tensor, $R = g^{\mu\nu} R_{\mu\nu}$ is curvature scalar, $g_{\mu\nu}$ is metric tensor, $T_{\mu\nu}$ is energy momentum tensor. By solving the equation of gravity field, the form of $g_{\mu\nu}$ can be obtained. The four-dimensional arc element is $ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$. In the gravitational field with spherical symmetry, the arc element is represented by the Schwarzschild Metric

$$ds^2 = c^2 \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \tag{11}$$

Here $\alpha = 2GM / c^2$. Let

$$A(r) = 1 - \frac{\alpha}{r} \qquad\qquad B(r) = \left(1 - \frac{\alpha}{r}\right)^{-1} \tag{12}$$

Substituting (11) in the geodesic equations of Riemann geometry, four equations are obtained. By Taking $\theta = \pi / 2$, one of them is zero. Remaining three are as follows 【2】,【4】

$$\frac{d^2 r}{ds^2} + \frac{B'}{2B}\left(\frac{dr}{ds}\right)^2 - \frac{r}{B}\left(\frac{d\varphi}{ds}\right)^2 + \frac{A'}{2B}\left(\frac{dx^0}{ds}\right)^2 = 0 \tag{13}$$

$$\frac{d^2 \varphi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds} = 0 \tag{14}$$

$$\frac{d^2 x^0}{ds^2} + \frac{A'}{A}\frac{dr}{ds}\frac{dx^0}{ds} = 0 \qquad 或 \qquad \frac{d}{ds}\left(A \frac{dx^0}{ds}\right) = 0 \tag{15}$$

Here $x^0 = ct$. The integrals of (14) and (15) are

$$r^2 \frac{d\varphi}{ds} = J \tag{16}$$

$$\frac{dx^0}{ds} = \frac{K}{A(r)} \tag{17}$$

In which J and K are integral constants.

3. 2 The deduction of motion equations of planets of general relativity

Because (13) is too complete, a simplified method using (11) to replace (13) is used to deduce the motion equation of planet in general relativity. Document [2] claimed that the result was the same but no strict poof was provided. We will prove later that the result is different. What obtained is not the real solution for the particle’s motion along the geodesic.

Taking $\theta = \pi / 2$ in (11), we write it as 【2】,【4】

$$\left(1 - \frac{\alpha}{r}\right)^{-1}\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 - \left(1 - \frac{\alpha}{r}\right)\left(\frac{dx^0}{ds}\right)^2 = -1 \tag{18}$$

Substituting (17) in (18) and considering (12), we get

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 = -1 + K^2 + \frac{\alpha}{r} + \frac{\alpha J^2}{r^3} \tag{19}$$

According to general relativity, by considering the Newtonian approximation to take $ds = cdt$ in (19), we get

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2(-1 + K^2) + \frac{2GM}{r} + \frac{\alpha c^2 J^2}{r^3} \tag{20}$$

By neglecting the item containing r^{-3} on the right side of (20) and comparing it with (8), we can determine the constant with $K^2 = 1 - GM/(c^2 b)$. Thus, (20) becomes

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = -\frac{GM}{b} + \frac{2GM}{r} + \frac{\alpha c^2 J^2}{r^3} \tag{21}$$

By neglecting the third item on the right side, (21) is the equation of ellipse orbit. So at present, the Newtonian theory of gravity is considered as the approximation of general relativity under weak condition. Substituting (16) in (18), we get

$$\left(\frac{dr}{ds}\right)^2 + \frac{J^2}{r^2} = -1 + K^2 + \frac{\alpha}{r} + \frac{\alpha J^2}{r^3} \tag{22}$$

Let $ds = cdt$ approximately and take derivative with respect to dt, (22) becomes

$$\frac{d^2 r}{dt^2} - \frac{c^2 J^2}{r^3} = -\frac{c^2\alpha}{2r^2} - \frac{3c^2\alpha J^2}{r^4} = -\frac{GM}{r^2} - \frac{3c^2\alpha J^2}{r^4} \tag{23}$$

Comparing it with (5), we can take $c^2 J^2 = L^2$. By considering (16) and eliminating ds in (22), the orbit equation which is unrelated to time is

$$\left(\frac{dr}{d\varphi}\right)^2 + r^2 = (-1 + K^2)\frac{r^4}{J^2} + \frac{\alpha r^3}{J^2} + \alpha r \tag{24}$$

Let $u = 1/r$, (24) becomes

$$\left(\frac{du}{d\varphi}\right)^2 + u^2 = \frac{-1 + K^2}{J^2} + \frac{\alpha}{J^2}u + \alpha u^3 \tag{25}$$

By derivative with respect to $d\varphi$ and let $c^2 J^2 = L^2$, (25) becomes

$$\frac{d^2 u}{d\varphi^2} + u = \frac{\alpha}{2J^2} + \frac{3\alpha u^2}{2} = \frac{GM}{L^2} + \frac{3GM}{c^2}u^2 \tag{26}$$

The second item on the right side is the revision item of general relativity. The most important verification of general relativity, the calculation of perihelion precession of Mercury, is based on (26). However, it should note that (26) can not yet determine what the concrete forms of planet's orbits are, parabola, ellipse or hyperbole. It is determined by the total mechanism energy E_0 as shown in (9).

3. 3 The problem caused by using approximate formula $ds = cdt$

1. In the deduction above, the proximate formula $ds = cdt$ is used, but it is irrational. If $ds = cdt$, two sides of (11) become unequal with ($\theta = \pi/2$)

$$c^2dt^2 \neq c^2\left(1-\frac{\alpha}{r}\right)dt^2 - \left(1-\frac{\alpha}{r}\right)^{-1}dr^2 - r^2d\varphi^2 \tag{27}$$

In this way, we can not get (18). All formulas below (18) become untenable.

2. In order to make (27) tenable, we should take

$$-\frac{c^2\alpha}{r}dt^2 = \left(1-\frac{\alpha}{r}\right)^{-1}dr^2 + r^2d\varphi^2 \tag{28}$$

The right side of (28) is negative but the left side is positive (when $r > \alpha$), so it is obviously impossible, unless $dt = dr = d\varphi = 0$. It means that objects have not displacement and no time process. In the theory of planet's motions, this is meaningless. So the condition $ds = cdt$ is completely impossible.

3. In fact, if $ds = cdt$, the metric is only relative to time, having nothing to do with space coordinates, the geodesic equation s (13) ~ (17) do not exist. It means that if the arc length is used to establish the geodesic equation, self-entanglement and serious contradiction will be caused when the motion equations of general relativity are deduced

3. 4 The correct calculation to use approximate formula

By consider the Newtonian approximate condition $\alpha / r << 1$ and $V / c << 1$, (11) can be written as ($\theta = \pi / 2$)

$$ds^2 = c^2\left[\left(1-\frac{\alpha}{r}\right) - \left(1-\frac{\alpha}{r}\right)^{-1}\left(\frac{dr}{cdt}\right)^2 - r^2\left(\frac{d\varphi}{cdt}\right)^2\right]dt^2$$

$$\approx c^2\left(1-\frac{\alpha}{r}-\frac{V_r^2}{c^2}-\frac{V_\varphi^2}{c^2}\right)dt^2 \tag{29}$$

According to (9), for the motion of parabola with $E_0 = 0$, we have

$$\frac{V_r^2}{c^2}+\frac{V_\varphi^2}{c^2}=\frac{\alpha}{r} \qquad ds^2 \approx c^2\left(1-\frac{2\alpha}{r}\right)dt^2 \tag{30}$$

For the motion of ellipse orbit with $E_0 = -GMm/2b$, we have

$$\frac{V_r^2}{c^2}+\frac{V_\varphi^2}{c^2}=\frac{\alpha}{r}-\frac{GM}{c^2b} \qquad ds^2 \approx c^2\left(1-\frac{2\alpha}{r}+\frac{GM}{c^2b}\right)dt^2 \tag{31}$$

For the motion of hyperbole orbit with $E_0 = +GMm/2a$, we have

$$\frac{V_r^2}{c^2}+\frac{V_\varphi^2}{c^2}=\frac{\alpha}{r}+\frac{GM}{c^2a} \qquad ds^2 \approx c^2\left(1-\frac{2\alpha}{r}-\frac{GM}{c^2a}\right)dt^2 \tag{32}$$

However, (29) means $B(r) = 1$ in (12), the Schwarzschild metric becomes ($\theta = \pi / 2$)

$$ds^2 = c^2\left(1-\frac{\alpha}{r}\right)dt^2 - dr^2 - r^2d\varphi^2 \tag{33}$$

Corresponding, we should take $B(r) = 1$ in the geodesic equation (13), thought (16) and (17) are unchanged. In this way, (18) becomes

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 - \left(1-\frac{\alpha}{r}\right)\left(\frac{dx^0}{ds}\right)^2 = -1 \tag{34}$$

Substituting (17) in (34) and considering $\alpha / r << 1$, we get

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 = -1 + K^2\left(1-\frac{\alpha}{r}\right)^{-1} = -1 + K^2 + \frac{K^2\alpha}{r} \tag{35}$$

Comparing (19) with (35), the revised item $\alpha J^2 / r^3$ of general relativity becomes $K^2\alpha^2 / r^2$.

Repeating previous procedure to substitute (16) in (35), let $u = 1/r$, $c^2 J^2 = L^2$ and $K^2 = 1$, then to take derivative with respective to $d\varphi$ again, the orbit equation unrelated to time is obtained

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{L^2} + \frac{2\alpha GM}{L^2}u \tag{36}$$

The second item in the right side of (36) is not the revised item of current general relativity. Using (36) to calculate the perihelion precession of Mercury, we can not get the result of 43'' a century. Only by this result, the approximation of (29) should be refused.

To obtain energy integral, for the motion of parabola, ds shown in (30) is submitted in (35), we get

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2\left(-1 + K^2 + \frac{K^2\alpha}{r}\right)\left(1-\frac{2\alpha}{r}\right)$$

$$= c^2(-1 + K^2) + \frac{c^2(2-K^2)\alpha}{r} - \frac{2c^2K^2\alpha^2}{r^2} \tag{37}$$

By neglecting the item containing α^2 / r^2 in (37), we get

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2(-1 + K^2) + \frac{(2-K^2)2GM}{r} \tag{38}$$

In the Newtonian theory of gravity, no matter what is the orbit, parabola, ellipse or hyperbole, the quantity in the front of $2GM / r$ should be equal to 1. Let $2 - K^2 = 1$, we have $K^2 = 1$. Substituting this result in (38) and comparing with (9), we get the equation of parabola with $E_0 = 0$. The result is coordinate. In this way, (37) becomes

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = \frac{2GM}{r} - \frac{2c^2\alpha^2}{r^2} \tag{39}$$

The revised item $\alpha L^2 / r^3$ of general relativity in (21) becomes $-2c^2\alpha^2 / r^2$ in (39). Let's discuss their difference. By taking $cJ = L \sim Vr$, under the Newtonian approximation with $V^2 / c^2 << 1$ and $2GM / r \sim V^2 \sim L^2 / r^2$, we have

$$-\frac{2c^2\alpha^2}{r^2} = -\frac{2GM}{r}\frac{2\alpha}{r} \sim -\frac{2\alpha V^2}{r} = -\frac{2\alpha L^2}{r^3} \tag{40}$$

Thought they have the same magnitude, $2c^2\alpha^2 / r^2$ is twice as $\alpha L^2 / r^3$, besides there is a difference of minus. Under strong field, the difference will be greater.

For the motion of ellipse orbit, ds shown in (31) is submitted in (35), we get

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2\left(-1+K^2+\frac{K^2\alpha}{r}\right)\left(1-\frac{2\alpha}{r}+\frac{GM}{c^2b}\right)$$

$$= c^2\left[-1-\frac{GM}{c^2b}+K^2\left(1+\frac{GM}{c^2b}\right)\right]+\left[2-K^2\left(1+\frac{GM}{c^2b}\right)\right]\frac{c^2\alpha}{r}-\frac{2c^2\alpha^2}{r^2} \qquad (41)$$

Under the Newtonian approximation, we neglect the item contain α^2/r^2 in (41) and get

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2\left[-1-\frac{GM}{c^2b}+K^2\left(1+\frac{GM}{c^2b}\right)\right]+\left[2-K^2\left(1+\frac{GM}{c^2b}\right)\right]\frac{2GM}{r} \qquad (42)$$

Similarly, for the Newtonian theory of gravity, the quantity before the item $2GM/r$ should be 1. By considering $GM/(2c^2b) << 1$, we have

$$2-K^2\left(1+\frac{GM}{c^2b}\right)=1 \qquad \text{or} \qquad K^2 = 1-\frac{GM}{c^2b} \qquad (43)$$

Substitute (43) in (42), we get

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = -\frac{GM}{b}+\frac{2GM}{r} \qquad (44)$$

The formula describe the ellipse motion, but (41) becomes

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = -\frac{GM}{b}+\frac{2GM}{r}-\frac{2c^2\alpha^2}{r^2} \qquad (45)$$

Comparing (45) with (21), the revised item $\alpha L^2/r^3$ of general relativity also becomes $-2c^2\alpha^2/r^2$. Under the strong field, the difference also becomes greater.

Similarly, Under the Newtonian approximation, for the motion of hyperbole orbit, according to (35), the revised equation is

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = \frac{GM}{c^2a}+\frac{2GM}{r}-\frac{2c^2\alpha^2}{r^2} \qquad (46)$$

The revised item $\alpha L^2/r^3$ of general relativity also becomes $-2c^2\alpha^2/r^2$.

In all calculations above, the approximate method is used and some uncertain is introduced. In order to obtain the precise equations of planet's motion, the arc length can not be used to establish geodesic equation. Besides, another obvious problem is that from (17) we get

$$ds^2 = \frac{c^2}{K^2}\left(1-\frac{\alpha}{r}\right)^2 dt^2 \qquad (47)$$

This result completely contradicts with the Schwarzschild metric (11). However, physicists did not pay attention to it up to now.

In the deduction above, the geodesic equation (13) has not been used. We use the Schwarzschild metric (11) to replace it. However, as we known, the Schwarzschild metric describe the distance between arbitrary two points in the space of gravitational field, rather than the arc length along the geodesic in general situation. But according to Riemann geometry and general relativity, the geodesic describe the motion orbits of particle in gravitational field. So the motion equation based on the (11) is not real one. The real equation should be deduced based on (13). If (13) is used, the result may be different in general. Especially, the approximate condition (29) is used, corresponding to $B(r)=1$ and $B'(r)=0$ in (31), we can not obtain the motion equations of general relativity.

3.5 The geodesic equation of Riemann geometry

According to general relativity, space-time was curved when material existed. By solving the Einstein's equation of gravity, the metric tensors which described the curvature of space-time were obtained written in the form of arc length. However, the curvature of space-time can not be measured directly. We can only reveal it in the form of particle's motion orbit in gravitational field. According to Riemann geometry, particles move along the geodesic in curved space.

Therefore, general relativity needs two set of equations to describe particle's motion in gravitational field. One is the Einstein's equation of gravity and another is the geodesic of Riemann geometry. Geodesic equation describes a special orbit on curved space, but the square of arc length describes whole nature of curved space. They are different in general situations. The square of arc length in Riemann geometry is

$$ds^2 = g_{jk}(x)dx^j dx^k \tag{53}$$

Here g_{jk} is the metric tensor related to space coordinates but unrelated to time. If space coordinates are considered as the functions of arc length, i.e., let $x^j = x^j(s)$, the arc length can be written in the form of integral

$$s = \int_{S_1}^{s_2} ds = \int_{S_1}^{s_2} \sqrt{g_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}}ds = \int_{S_1}^{s_2} H(x^j,\dot{x}^j)ds \tag{54}$$

The variation method is used to obtain the extreme value of function $H(x^j,\dot{x}^j)$. The Euler's equation is

$$\frac{\partial H}{\partial x^j} - \frac{\partial}{\partial s}\frac{\partial H}{\partial(\partial \dot{x}^j)} = 0 \tag{55}$$

From (55), the geodesic equation is obtain

$$\frac{d^2x^j}{ds^2} + \Gamma^j_{\sigma\rho}\frac{dx^\sigma}{ds}\frac{dx^\rho}{ds} = 0 \tag{56}$$

Here $\Gamma^j_{\sigma\rho}$ is the Christopher's sign, composed of metric tensor and their derivative with respected to space coordinates. By solving the geodesic equations, a set of relations is obtained. For example, there are three geodesic equations in three-dimensional space. Their solutions are $x^1 = x^1(s)$, $x^2 = x^2(s)$ and $x^3 = x^3(s)$. By eliminating parameter s in them, two relations $f_1(x^1,x^2)=0$ and $f_2(x^2,x^3)=0$ are obtained which describe two curved surfaces. Their intersecting line is just the geodesic.

However, as we known that the variation function should be a variable when the calculus of variation method is used to find the extreme values. Otherwise, there is no extreme value. So (56) has a fatal problem. If the formula is tenable, we should have【6】:

$$H(x^j,\dot{x}^j) = \sqrt{g_{jk}\frac{\partial x^j}{\partial s}\frac{\partial x^k}{\partial s}} = 1 \tag{57}$$

It means that H is a constant. Substitute it in (55), the result should be zero so that geodesic equations can not be obtained. So in the Gaussian differential geometry, the square of arc length is not written as

ds^2. It is represented by following formula, called the first basic form of curved surface.

$$I = E(u,v)du^2 + F(u,v)dudv + G(u,v)dv^2 \tag{58}$$

Then, the arc length s is used as variation parameter to establish geodesic equation (56). In this way, thought the contradiction is avoided superficially, the problem has not been solved really. In differential geometry, the arc length can not be used to construct geodesic. This problem is very foundational, but mathematicians seem to pay no attention to it.

In fact, the arc length s is only a variation parameter in geodesic equation. We can use other quantity replace it and get the same result. Because ds does not contain time in the pure space geometry, we can use time t as variation parameter to rewrite (54) as below

$$s = \int_{S_1}^{s_2} ds = \int_{t_1}^{t_2} \sqrt{g_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt}} dt = \int_{S_1}^{s_2} H(x^j, \dot{x}^j) dt \tag{59}$$

Accordingly, s in (56) is also replaced by t. After the integral of geodesic, we get three formulas $x^1 = x^1(t)$, $x^2 = x^2(t)$ and $x^3 = x^3(t)$. By eliminating t in them, we obtain $F_1(x^1, x^2) = 0$ and $F_2(x^2, x^3) = 0$. The geodesic is determined by these two relations.

In general relativity, space and time coordinates can not be separated. The four-dimensional arc length ds contains time. We can not use time as variation parameter, but can used a arbitrary parameter ζ and write geodesic equation as

$$\frac{d^2 x^i}{d\zeta^2} + \Gamma^i_{jk} \frac{dx^j}{d\zeta} \frac{dx^k}{d\zeta} = 0 \tag{60}$$

The formula (60) and its solution do not contain arc length. Based on it, we can discuss the motion equation of general relativity strictly.

3.6 The geodesic equation of Schwarzschild metric using arbitrary variation parameter

By using arbitrary variation parameter, and based on the Schwarzschild metric (11), the geodesic equation become

$$\frac{d^2 r}{d\zeta^2} + \frac{B'}{2B}\left(\frac{dr}{d\zeta}\right)^2 - \frac{r}{B}\left(\frac{d\varphi}{d\zeta}\right)^2 + \frac{A'}{2B}\left(\frac{dx^0}{d\zeta}\right)^2 = 0 \tag{61}$$

$$\frac{d^2\varphi}{d\zeta^2} + \frac{2}{r}\frac{dr}{d\zeta}\frac{d\varphi}{d\zeta} = 0 \tag{62}$$

$$\frac{d^2 x^0}{d\zeta^2} + \frac{A'}{A}\frac{dr}{d\zeta}\frac{dx^0}{d\zeta} = 0 \quad \text{or} \quad \frac{d}{d\zeta}\left(A\frac{dx^0}{d\zeta}\right) = 0 \tag{63}$$

The integrals of (62) and (63) are

$$\frac{dx^0}{d\zeta} = \frac{K}{A(r)} \tag{64}$$

$$r^2 \frac{d\varphi}{d\zeta} = J \tag{65}$$

It notes that the forms of (61) ~ (65) are completely the same as (13) ~ (17), except to use ζ to replace s. But $d\zeta$ does not need to satisfy (11), there is no trouble again. Substituting (64) and (65) in (61), we get【4】

$$\frac{d^2r}{d\zeta^2}+\frac{B'}{2B}\left(\frac{dr}{d\zeta}\right)^2-\frac{J^2}{Br^3}+\frac{A'K^2}{2BA^2}=0 \tag{66}$$

(66) can be written as [4]

$$\frac{d}{d\zeta}\left[B\left(\frac{dr}{d\zeta}\right)^2+\frac{J^2}{r^2}-\frac{K^2}{A}\right]=0 \tag{67}$$

The integral of（67）is

$$B\left(\frac{dr}{d\zeta}\right)^2+\frac{J^2}{r^2}-\frac{K^2}{A}=-E \tag{68}$$

In which E is an integral constant. From (68), we have

$$\frac{dr}{d\zeta}=\frac{1}{B^{1/2}}\sqrt{-E-\frac{J^2}{r^2}+\frac{K^2}{A}} \tag{69}$$

The formulas (64)，(65) and (67) are the first integrals of geodesic equations. Substituting them in (11), we get

$$ds^2=Ed\zeta^2 \qquad \text{or} \qquad s=\sqrt{E\zeta^2+b} \tag{70}$$

Here b is a integral constant. This is just the relation between the arc length and variation parameter in the gravitational field with spherical symmetry. If only substituting (64) and (65) in (11), we can not get (70). In this case, ds does not represent the arc length of geodesic.

3.7 The strict proof that general relativity can only describe parabola orbit

Substituting (65) in (66) and eliminating $d\zeta$, we get [4]

$$\frac{B}{r^4}\left(\frac{dr}{d\varphi}\right)^2+\frac{1}{r^2}-\frac{K^2}{AJ^2}=-\frac{E}{J^2} \tag{71}$$

Let $u=1/r$ and consider (12), we get

$$\left(\frac{du}{d\varphi}\right)^2+u^2=\frac{K^2-E}{J^2}+\frac{E\alpha}{J^2}u+\alpha u^3 \tag{72}$$

Taking derivative (72) with respect to $d\varphi$ again, we have

$$\frac{d^2u}{d\varphi^2}+u=\frac{E\alpha}{2J^2}+\frac{3}{2}\alpha u^2=\frac{GME}{L^2}+\frac{3GM}{c^2}u^2 \tag{73}$$

Omitting the second item on the right side of equal sign, and take $E=1$, (73) is just the formula of Newtonian gravity. As for what kind of orbit it describes, parabola, ellipse or hyperbole, we need to do calculation further. It depends on integral constants K and E. We discuss this problem below. From (64) and (65), we have

$$\frac{d\varphi}{dt}=\frac{cAJ}{Kr^2} \tag{74}$$

Substituting (12), (64) and (74) in (68) to eliminate $d\zeta$, we get

$$\left(\frac{dr}{dt}\right)^2+r^2\left(1-\frac{\alpha}{r}\right)\left(\frac{d\varphi}{dt}\right)^2=c^2\left(1-\frac{\alpha}{r}\right)^2-\frac{c^2E}{K^2}\left(1-\frac{\alpha}{r}\right)^3 \tag{75}$$

By using (74) again, we obtain

$$\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2=c^2\left(1-\frac{\alpha}{r}\right)^2-\frac{c^2E}{K^2}\left(1-\frac{\alpha}{r}\right)^3+\alpha r\left(\frac{d\varphi}{dt}\right)^2$$

$$=c^2\left(1-\frac{E}{K^2}\right)+c^2\left(\frac{3E}{K^2}-2\right)\frac{\alpha}{r}+c^2\left(1-\frac{3E}{K^2}\right)\frac{\alpha^2}{r^2}+\frac{c^2E}{K^2}\frac{\alpha^3}{r^3}+\frac{\alpha c^2J^2}{K^2r^3}\left(1-\frac{\alpha}{r}\right)^2 \tag{76}$$

Under the Newtonian approximation, the third, fourth and fifth items on the right side can be neglected, (76) becomes

$$\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2=c^2\left(1-\frac{E}{K^2}\right)+\left(\frac{3E}{K^2}-2\right)\frac{2GM}{r} \tag{77}$$

For the motion equation of the Newtonian gravity, the parameter in the front of item $2GM/r$ should be equal to 1. Let $3E/K^2-2=1$, we have $E/K^2=1$. So the constant item in the right side of (77) is zero, the equation can only describe the orbit of parabola. Beyond that, we have no other choose. By considering higher order items, (76) is

$$\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2=\frac{2GM}{r}-\frac{2c^2\alpha^2}{r^2}+\frac{c^2\alpha^3}{r^3}+\frac{\alpha c^2J^2}{K^2r^3}\left(1-\frac{\alpha}{r}\right)^2$$

$$\approx\frac{2GM}{r}-\frac{2c^2\alpha^2}{r^2}+\frac{\alpha c^2J^2}{K^2r^3} \tag{78}$$

Comparing it with (20) and taking $K^2=1$, there is the item $-2\alpha^2/r^2$ more in (78). As shown in (40), this item has the same magnitude with the third item of relativity revision under weak field condition. In general situation, it may be greater then the third item. So (78) is also different from the current general relativity. By taking derivation (78) with respect to dt, the motion equation obtained is also different from the current one in general relativity.

In this way, we prove strictly that general relativity can not describe the ellipse and hyperbole orbits of planets in the gravitational filed with spherical symmetry. It can only describe the parabola orbits. Because the ellipse orbit is the most foundational motion form of planets in the solar system, general relativity is invalid as a foundational theory of physics.

3.8 The motion equation of light in gravity field.

According to general relativity, light's motion in gravity field is described by $ds=0$. So we can not use (59) to construct the geodesic equation. The reason is that, similar to the principle of least action in optics, $ds=0$ has represent the minimum value of arc length. The variation calculation on the relation $ds=0$ means that the second order derivative of a function is zero. The result is used to judge the inflection point, in stead of the extreme value of function.

General relativity also uses (61) ~ (63) as the geodesic equations of light, but takes $E = 0$ in the formulas. However, for a curved space, the form of geodesic is unique, having nothing to do with what kind of particle moves along the geodesic. That is to say, according to Riemann geometry, no mater whether particle has mass, no mater how much particle's mass is, when they move along the geodesic in the curved space, the equations of geodesics should be the same.

According to (70), for a particle with mass, we take $E \neq 0$. The arc length $s = \sqrt{E\zeta + b}$ does not equal to constant along geodesic. But for a photon with zero mass, we take $E = 0$. The arc length becomes $s = \sqrt{b}$. That is to say, the distance between two points on the geodesic is different. This violated the basic principle of Riemann geometry.

By considering the unique of geodesic in curved space, we should use (64) (65) and (68) to describe light's motion in gravity field. Substituting (65) in (66) and eliminating $d\zeta$, we get (73). Let $E = 0$ in (73) we get

$$\frac{d^2u}{d\varphi^2} + u = \frac{3}{2}\alpha u^2 \qquad (79)$$

According to (70), we get $ds = d\sqrt{b} = 0$. The theory seems consistent. However, the problem is not so simple. Form (64) and (65), we get (74). Substituting （12）、（64） and （74） in （68） and eliminating $d\zeta$, we get (76). Let $E = 0$ in (76)， we get below result

$$\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = c^2\left[1 - \frac{2\alpha}{r} + \frac{\alpha^2}{r^2} + \frac{\alpha J^2}{K^2 r^3}\left(1 - \frac{\alpha}{r}\right)^2\right] \qquad (80)$$

Let $cJ = L$, (80) can be written as the formula of energy conservation similar to that in the Newtonian theory

$$\frac{1}{2}m_0\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right] + \frac{2GMm_0}{r}\left[1 - \frac{\alpha}{r} - \frac{L^2}{2K^2c^2r^2}\left(1 - \frac{\alpha}{r}\right)^2\right] = \frac{1}{2}m_0c^2 \qquad (81)$$

Here $m_0c^2/2 = h\nu$ can be considered as photon's energy outside gravitational field. The corresponding static mass of photon is $m_0 = 2h\nu/c^2$. Comparing with the form of Newtonian gravity, we get following conclusions.

1. The revision of general relativity is corresponding to define the gravitational mass of photon as

$$m_g = m_0\left[1 - \frac{\alpha}{r} - \frac{L^2}{2K^2c^2r^2}\left(1 - \frac{\alpha}{r}\right)^2\right] \qquad (82)$$

2. The gravity energy of photon is positive. It indicates that light is acted by repulsive force in gravitational field.

3. Because light's energy is positive, so the orbit of light is hyperbole in gravitational field with spherical symmetry.

When a photon moves along the radius direction in the solar system with $L = 0$, (81) becomes

$$\frac{1}{2}m_0\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2\right] + \frac{2GMm_0}{r}\left(1 - \frac{\alpha}{r}\right) = \frac{1}{2}m_0c^2 \qquad (83)$$

The gravity mass of photon is twice of its inertial mass. In the weak field with $\alpha/r << 1$, light's speed and acceleration are

$$V \approx c\sqrt{1-\frac{2\alpha}{r}} \qquad\qquad a=\frac{dV}{dt}=\frac{dV}{dr}\frac{dr}{dt}=\frac{c\alpha}{r^2}\geq 0 \tag{84}$$

If photon moves along the tangential direction of solar surface, at the pole with $L=VR$, (81) becomes

$$\frac{1}{2}m_0\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2\right]+\frac{2GMm_0}{r}\left[1-\frac{\alpha}{r}-\frac{V^2R^2}{2c^2K^2r^2}\left(1-\frac{\alpha}{r}\right)^2\right]=\frac{1}{2}m_0c^2 \tag{85}$$

By considering the Newtonian approximation with $\alpha/r<<1$ and taking $K^2=1$. At the points nearing the pole , $r\approx R$, (81) becomes

$$\frac{1}{2}m_0\left[\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\varphi}{dt}\right)^2\right]+\frac{GMm_0}{r}=\frac{1}{2}m_0c^2 \tag{86}$$

The gravity mass of photon becomes the same as its inertial mass. Light's speed and acceleration are

$$V=c\sqrt{1-\frac{\alpha}{r}} \qquad\qquad a=\frac{dV}{dt}=\frac{dV}{dr}\frac{dr}{dt}=\frac{c^2\alpha}{2r^2}=\frac{GM}{r^2}\geq 0 \tag{87}$$

Photon is acted by repulsive force in gravitational field. The result is quite strange. We will discuss these problems in another paper and prove that the motion equation of general relativity has serious problems so that it is impossible to be used to describe light's motion in the solar gravity field 【5】.

4. The deformations of planet's orbits in general relativity

4. 1 The calculation of general relativity on the perihelion precession of Mercury

Now we do not consider that general relativity can not describe the ellipse orbit of planet. According to general, relativity, the motion equation of planets can be written as

$$\frac{d^2u}{d\varphi^2}+u=\frac{GM}{c^2h^2}+\frac{3GM}{c^2}u^2 \tag{88}$$

By neglecting the second item on the right side and taking the integral, the ellipse orbit of planet is

$$u_1=\frac{GM}{c^2h^2}(1-e\cos\varphi) \tag{89}$$

As the second order approximation, substitute (89) on the right side of (88) and take advantage of formula $\cos^2\varphi=(1+\cos 2\varphi)/2$, we have 【1】

$$\frac{d^2u}{d\varphi^2}+u=\frac{GM}{c^2h^2}\left[1+\frac{3G^2M^2}{c^4h^2}-\frac{6G^2M^2}{c^4h^2}e\cos\varphi+\frac{3G^2M^2e^2}{2c^4h^2}(1+\cos 2\varphi)\right] \tag{90}$$

On the other hand, we consider following equation

$$\frac{d^2u_2}{d\varphi^2}+u_2=A\cos\varphi \tag{91}$$

Its solution is

$$u_2=\frac{1}{2}A\varphi\sin\varphi \tag{92}$$

By considering $G^2M^2/(c^4h^2) \sim 10^{-7} << 1$ and (92), omitting the second and forth items in (90), the second order approximate solution of（88）is 【1】

$$u = u_1 + u_2 = \frac{GM}{c^2h^2}\left[1 - e\cos\varphi - \frac{3G^2M^2}{c^4h^2}e\varphi\sin\varphi\right] \qquad (93)$$

This is a non-period solution in which the factor containing φ will accumulate and causes observable effects. Let $\delta = 3G^2M^2/(c^4h^2)$, we have $\delta \approx 10^{-7} << 1$, $\delta\cdot\varphi << 1$, $\sin\delta\cdot\varphi = \delta\cdot\varphi$ and $\cos\delta\cdot\varphi = 1$.（93） can be written as

$$u = \frac{GM}{c^2h^2}\left[1 - e\cos\varphi - e\delta\cdot\varphi\sin\varphi\right] = \frac{GM}{c^2h^2}\left[1 - e\cos(1-\delta)\varphi\right] \qquad (94)$$

The perihelion precession value for the Mercury to move around the sun a circle is

$$\Delta\varphi = 2\pi\times\delta\varphi = \frac{6\pi G^2M^2}{c^4h^2} = \frac{6\pi GM}{c^2b(1-e^2)} = 5.0197\times 10^{-7} \qquad (95)$$

The time for the Mercury moving around the sun a circle is about 88 earth days, so the precession value of the Mercury is $415.2\times 5.0197\times 10^{-7} = 43''$ a century.

4. 2 The change of semi-latus rectum of Mercury orbit

The physical signification of (93) is that a non-period solution is added in the period solution of ellipse orbit. According to (93), not only the revision of general relativity causes the precession, but also cause the changes of orbit's shapes. The changes of orbit's shapes are far great than that of rotating angle and more easy to be observed. Comparing with the former, the later becomes insignificant and can be neglected.

Let us do concrete calculation. take $\varphi = \pi/2$ in (93), we have

$$u = \frac{GM}{c^2h^2}\left[1 - \frac{3G^2M^2e\pi}{2c^4h^2}\right] \qquad (96)$$

As shown in Fig. 1, according to（89）, when $\varphi = \pi/2$, $r = p = GM/(c^2h^2) = b(1-e^2)$, p is the semi -latus rectum of planet's ellipe orbit. For Mercury, $b = 5.7911\times 10^{10}$, $e = 0.20561$, we have

$$\frac{3G^2M^2e\pi}{2c^4h^2} = \frac{3G^2M^2e\pi}{2c^4GMb(1-e^2)/c^2} = \frac{3GMe\pi}{2c^2b(1-e^2)} = 2.5893933367\times 10^{-8} \qquad (97)$$

$$\frac{GM}{c^2h^2} = \frac{3GM}{c^2GMb(1-e^2)/c^2} = \frac{1}{b(1-e^2)} = 1.8030107892\times 10^{-11} \qquad (98)$$

Substituting them in (96) , we obtain

$$u = 1.8030107892\times 10^{-11}(1 - 2.5893933367\times 10^{-8})$$

$$r = \frac{1}{u} = 5.5462785152\times 10^{10}(1 + 2.5893933367\times 10^{-8})$$

$$= 55462785152 + 1436 \text{ m} \qquad (99)$$

（99）indicates that the semi-latus rectum increases 14436m after Mercury moves $\pi/2$ angle around the perihelion. After Mercury moves around the sun n times, the semi-latus rectum increases $1436\times n$ m. Mercury moves around the sun about 4 times in an earth year, the increase of semi-latus rectum is 5744m a year. This is a great enough quantity to be observed, but we can not find it.

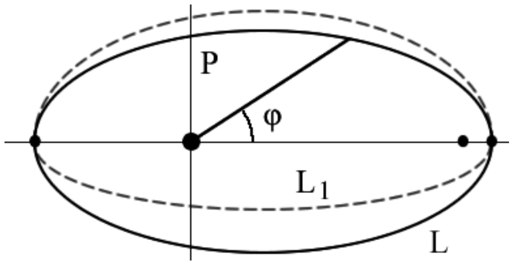

Fig. 1 The change of semi-latus rectum of Mercury orbit

Besides, as shown by imaginary line L_1 in Fig. 1, the change of semi-latus rectum is asymmetrical. After Mercury moves around the sun for $3\pi/2$ angle with $\sin 3\pi/2 = -1$, according to (93), we have

$$u = \frac{GM}{c^2h^2}\left[1 + \frac{9G^2M^2e\pi}{2c^4h^2}\right] \qquad (100)$$

$$r = \frac{1}{u} = 5.5462785152 \times 10^{10}(1 - 7.7681800101 \times 10^{-8}) = 55462785152 - 4308 \text{ m} \qquad (101)$$

The semi-latus rectum of orbit decreases 4308m. After Mercury moves around the sun n times, the semi-latus rectum decreases $4308 \times n$ m. Such great and asymmetrical change would cause serious deformation and destroys the orbit motion of Mercury. But no such deformation is observed for Mercury in astronomy.

4. 3 Using numerical method to calculate the perihelion precession of Mercury

The fourth item on the right side of (90) is neglected in the discussion of 5.1 Section. Corresponding equation and its special solution are

$$\frac{d^2u_3}{d\varphi^2} + u_3 = A\cos 2\varphi \qquad\qquad u_3 = -\frac{1}{3}A\cos 2\varphi \qquad (102)$$

By considering the effect of this item, (93) becomes

$$u = \frac{GM}{c^2h^2}\left[1 - e\cos\varphi - \frac{3G^2M^2e}{c^4h^2}\left(\varphi\sin\varphi - \frac{e}{2}\cos 2\varphi\right)\right] \qquad (103)$$

By derivative (103) with respect to angle φ and let the result to be zero, we get following equation to calculate the poles of ellipse orbit for the new perihelion and aphelion of Mercury.

$$\sin\varphi - 8.0173879761 \times 10^{-8}(\sin\varphi + \varphi\cos\varphi + 0.20561\sin 2\varphi) = 0 \qquad (104)$$

Using the numerical method of computer to calculate zero point (error smaller than 10^{-30}), we obtain following angles φ. All of them are nearby $n\pi$. For simplifications, we only write them out in ten digits.

$$\varphi_1 = 3.1415929056 \qquad \varphi_2 = 6.2831858109 \qquad \varphi_3 = 9.4247787164$$

$$\varphi_4 = 12.5663716219 \qquad \varphi_5 = 15.70796452 \qquad \varphi_5 = 18.8495574328 \qquad (105)$$

The changes of angles are the precessions of perihelion or aphelion when Mercury moves around the sun a circle. Taking $\pi = 3.1415926536$, the results are

$$\varDelta\varphi_1 = 2(\varphi_1 - \pi) = 5.0374736736 \times 10^{-7} \qquad \varDelta\varphi_2 = 2(\varphi_2 - \varphi_1 - \pi) = 5.0374740058 \times 10^{-7} \tag{106}$$

$$\varDelta\varphi_3 = 2(\varphi_3 - \varphi_2 - \pi) = 5.0374736736 \times 10^{-7} \qquad \varDelta\varphi_4 = 2(\varphi_4 - \varphi_3 - \pi) = 5.0374740058 \times 10^{-7} \tag{107}$$

$$\varDelta\varphi_5 = 2(\varphi_5 - \varphi_4 - \pi) = 5.0374736736 \times 10^{-7} \qquad \varDelta\varphi_6 = 2(\varphi_6 - \varphi_5 - \pi) = 5.0374740058 \times 10^{-7} \tag{108}$$

It indicates that when Mercury moves around the sum a circle, the precession value of angle is nearby 5.03747×10^{-7} rad. According to the calculation of general relativity, the value is 5.0102×10^{-7} rad. The consistence using two methods indicates the effectiveness of numerical calculation in this paper.

4. 4 The changes of perihelion and aphelion of Mercury orbit

Let $\varphi = 0$ in (93), we get

$$u_0 = \frac{GM}{c^2h^2}\left[1 - e + \frac{3G^2M^2e^2}{2c^4h^2}\right]$$

$$= 1.8030107892 \times 10^{-11}\left(0.79439 + 1.694694084 \times 10^{-9}\right)$$

$$= 1.4322937408\ 3 \times 10^{-11}(1 + 2.1333275645\ 4 \times 10^{-9}) \tag{109}$$

$$r_0 = 1/u_0 = 6.9783975429 \times 10^{10}(1 + 2.1333275645\ 4 \times 10^{-9})$$

$$= 6978397542\ 9 - 149 = 6978397528\ 0 \text{ m} \tag{110}$$

Let $\varphi = \varphi_1 = 3.1415929056$ in (93) with $\sin\varphi_1 = -0.0548036695\ 39$, $\cos\varphi_1 = -0.9984971496$ and $\cos 2\varphi_1 = 0.9939931156$, we get

$$u_1 = 1.8030107892 \times 10^{-11}\big(1 + 0.20561 \times 0.9984971496 - 1.6484551418 \times 10^{-8} \times$$

$$\times(-3.1415929055 \times 0.0548036695\ 39 + 0.102805 \times 0.9939931156)\big)$$

$$= 1.8030107892 \times 10^{-11}\big(1.2053009989 - 1.6484551418 \times 10^{-8}$$

$$\times(-0.1721708194 + 0.1021874622)\big)$$

$$= 1.8030107892 \times 10^{-11}\big(1.2053009989 - 1.6484551418 \times 10^{-8} \times 0.0699833572\big)$$

$$= 2.1731707053 \times 10^{-11}(1 - 9.5914203430 \times 10^{-10}) \tag{111}$$

$$r_1 = 1/u_1 = 4.6015712283 \times 10^{10}(1 + 9.5914203430 \times 10^{-10})$$

$$= 4601571228\ 3 - 44 = 4.6015712239 \times 10^{10} \text{ m} \tag{112}$$

Let $\varphi = \varphi_2 = 6.2831858109$ in (93) with $\sin\varphi_2 = 0.1094426156$, $\cos\varphi_2 = 0.9939931156$ and $\cos 2\varphi_1 = 0.9760446278$, we have

$$u_2 = 1.8030107892 \times 10^{-11}\big(1 - 0.20561 \times 0.9939931156 - 1.6484551418 \times 10^{-8}$$

$$\times(6.2831858109 \times 0.1094426156 - 0.102805 \times 0.9939931156)\big)$$

$$= 1.8030107892 \times 10^{-11}\big(0.7956250746 - 1.6484551418 \times 10^{-8}$$

$$\times(0.6876482894 - 0.1022321919)\big)$$

$$= 1.8030107892 \times 10^{-11}(0.7956250746 - 9.2650321760 \times 10^{-9})$$

$$= 1.4345205937 \times 10^{-11}(1 - 1.1644975278 \times 10^{-8}) \tag{113}$$

$$r_2 = 1/u_2 = 6.9709699839 \times 10^{10}(1 + 1.1644975278 \times 10^{-8})$$

$$= 6970969983\ 9 + 812 = 6.9709700651 \times 10^{10}\ \text{m} \qquad (114)$$

Let $\varphi = \varphi_3 = 9.4247787164$ in (93) with $\sin\varphi_3 = -0.1637526100$, $\cos\varphi_3 = -0.9865014357$ and $\cos 2\varphi_3 = 0.9463701654$, we get

$$u_3 = 1.8030107892 \times 10^{-11}\big(1 + 0.20561 \times 0.9865014357 + 1.6484551418 \times 10^{-8}$$

$$\times(-9.4247787164 \times 0.1637526100 - 0.102805 \times 0.9463701654)\big)$$

$$= 1.8030107892 \times 10^{-11}(1.2028345602 + 1.6484551418 \times 10^{-8} \times 1.4460405286)$$

$$= 1.8030107892 \times 10^{-11}(1.2028345602 + 2.3837329446 \times 10^{-8})$$

$$= 2.1687236896\ 7 \times 10^{-11}(1 + 1.9817629319 \times 10^{-8}) \qquad (115)$$

$$r_3 = 1/u_3 = 4.6110069473 \times 10^{10}(1 - 1.9817629319 \times 10^{-8})$$

$$= 4611006947\ 3 - 914 = 4.6110068559 \times 10^{10}\ \text{m} \qquad (116)$$

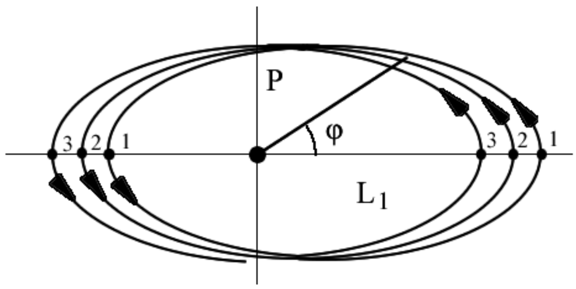

Fig. 2 The changes of perihelion and aphelion of Mercury orbit

The changes of perihelion and aphelion of Mercury orbit are shown in Fig.2. The precession angle of orbit is too small to be drawn out. By connecting Fig. 1 and Fig. 2, we obtain the motion model of the Mercury based on general relativity as shown below.

1. Mercury moves around the sun a circle, the decease of aphelion's distance is

$$r_2 - r_0 = 6970970065\ 1 - 6978397528\ 0 = -7.4274629 \times 10^{7}\ \text{m} \qquad (117)$$

The increase of perihelion's distance is

$$r_3 - r_1 = 4611006855\ 9 - 4601571223\ 9 = 9.4356320 \times 10^{7}\ \text{m} \qquad (118)$$

The values are great, due to the great cardinal numbers of orbit's radius. According to this speed of change, when Mercury moves around the sun 321circles (about 80 earth years) , aphelion would become perihelion. After Mercury moves around the sun 252 circles (about 63 earth years) , perihelion would become aphelion.

2. The changes of perihelion and aphelion are caused by the second item $e\cos\varphi$ on the right side of (93). The contribution of the third item $\varphi\sin\varphi$ is small. But for long time's accumulation, the effect of the third item will become great. For example, if Mercury moves around the sun one million circles (about 250 millenniums), the third item will reach 0.5 magnitude. The distance of perihelion would increase a half. However, 250 millenniums is only a moment for the process of the universe.

3. It is impossible for the mercury's orbit to have so great deformations. The observation of Mercury orbit motion can be traced back to 1765. Since that time, Mercury has moved around the sun for 1045 circles. According to calculations above, the perihelion and aphelion should have exchanged 3 times. But

no any can be found in astronomy observation. The revision of general relativity is untenable.

5. Discussions on the perihelion precession of Mercury

5. 1 The calculation of general relativity on the perihelion precession of Mercury is meaningless

Einstein used general relativity to calculate the perihelion precession of Mercury and obtained the result of 43” a century. This was considered as the most important achievement of general relativity. Due to this result, general relativity began to be accepted by scientific community. However, due to following three reasons, the calculation of general relativity is meaningless

1. According to the discussion above, because planets can not do the motions of ellipse, the perihelion precession is out of question.

2. The coarse method was used in the calculations of other planet’s perturbations on the motion of Mercury. The error may be far great than 43’’ a century (discussed in 5.2 section).

3. There are three direct proofs to prove that general relativity can not explain the precessions of binary stars.

I) General relativity can not explain the precession of DI Herkules. According to the analyses of Martynov D. I. & Khaliun K. F. based on 3000 orbit data in 1984, the precession value of DI Herkules was 0.64 degree. But according to the calculation of general relativity, the precession value is 2.34 degree. The error is great【6】.

II）For the A star of pulse PSRJ0737－3039A/B, the observation valve of precession is 75 degree/year. For the B star，the observation valve of precession is 71 degree/year. But according to general relativity, the precession valve is 14.7 degree/ year. The error is far greater.

III）For the pulse PSR1913+16, according to general relativity, the precession valve of pulse star is 2.68 degree/ year, the precession valve of compassion star is 2.49 degree/year. But practical observation is 4.2 degree /year.

5. 2 The calculating error caused by the Newtonian mechanics

The perihelion precession of Mercury was calculated before special relativity was born. LeVerrier pointed out in 1845 that by deducting other factors, there was a value of 35″ a century could not be explained for the perihelion precession of Mercury. Later, the value was revised as 43″ a century. However, practical observation valve was 5557′′ a century. In which 532′′ was caused by other planet’s perturbations and 4982″ was caused by the precession and nutation of the Earth reference frame. By deducting theses two values, remain was 43″ of general relativity【4】.

So 43″ is not the direct calculating result of general relativity. Thus, the problem was caused. Because the value 5514″ was calculated by the Newtonian theory, many errors would be contained. At first, wshould used general relativity to calculate the precession and the nutation of the Earth reference frame and get the same result of 4982″a century. Then use general relativity to calculate other planet’s perturbations and get the same result of 532′′ a century. These are obviously impossible. In fact, for these problems, we can not even write out the motion equation of general relativity, much less to calculate them.

More serious problem is that for the effects of other planets on the perihelion precession of Mercury, we can not use Newtonian theory to do accurate calculation. A coarse method was used so that the error may be great than 43″ a century. A comment in Wang Lingjun’s paper “One hundred years of general relativity—A critical view" is introduced below 【7】.

Because the motion of multi-body is involved, it is impossible to calculate the effects of other planets on the perihelion precession of Mercury precisely. Astrophysics used a simplified method to break other planets into uniform ring medium round the sun and calculated the effects on Mercury's motion. According to this simplified model, the theoretical value of Mercury's precession was 532" a hundred years. The calculating vale of general relativity was 43" a hundred years which was only 8% of 532". If the precession and nutation of the Earth reference frame was considered, the calculating vale 43" of general relativity was only 0.8% of 5557" a hundred years.

In the common textbook of general relativity, the error analyze was not considered. The general textbooks only said that general relativity explained the Mercury's precession of 43" a hundred years. It fact, the calculating error of Newtonian theory was far great than 10%. In this way, the calculation of general relativity was meaningless. The claim that general relativity has explained the perihelion precession of Mercury is dishonest and very easygoing.

So the true is that the perihelion precession of Mercury which needed to be considered is not 43" a hundred years. The calculation of general relativity is meaningless. It is proved below that many factors may affect the value of precession. For example, by considering special relativity, we can obtain the precession value of 14.4" a hundred years.

5. 3 The effect of special relativity on the perihelion precession of Mercury

The following method comes from reference【8】in which special relativity was used to calculate the orbit of an electron in hydrogen atom. By considering the equivalent principle between gravity mass and inertial motion mass, the motion equation of special relativity for a particle moving in gravitational field is

$$\frac{d}{dt}\left(\frac{m}{\sqrt{1-V^2/c^2}}\frac{d\vec{r}}{dt}\right)=-\frac{GMm}{\sqrt{1-V^2/c^2}}\frac{\vec{r}}{r^3} \qquad (119)$$

In column coordinate system, the equation becomes

$$\frac{d}{dt}\left(\frac{m\dot{r}}{\sqrt{1-V^2/c^2}}\right)-\frac{mr\dot{\varphi}^2}{\sqrt{1-V^2/c^2}}=-\frac{GMm}{r^2\sqrt{1-V^2/c^2}} \qquad (120)$$

$$\frac{d}{dt}\left(\frac{mr\dot{\varphi}}{\sqrt{1-V^2/c^2}}\right)=0 \qquad (121)$$

The integral of (121) is

$$\frac{mr^2\dot{\varphi}}{\sqrt{1-V^2/c^2}}=K \qquad (122)$$

Here K is a constant to represent angle momentum. Form (122), we get

$$dt=\frac{mr^2}{K\sqrt{1-V^2/c^2}}d\varphi \qquad (123)$$

Substituting (122) and (123) in (120), we get

$$\frac{K\sqrt{1-V^2/c^2}}{mr^2}\frac{d}{d\varphi}\left(\frac{1}{r^2}\frac{dr}{d\varphi}\right)-\frac{K^2\sqrt{1-V^2/c^2}}{mr^3}=-\frac{GMm}{r^2\sqrt{1-V^2/c^2}} \qquad (124)$$

By considering $V^2/c^2 << 1$, we have from (124)

$$\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right)+\frac{1}{r}=\frac{GMm^2}{K^2(1-V^2/c^2)}=\frac{GMm^2}{K^2}\left(1+\frac{V^2}{c^2}\right) \tag{125}$$

In the Newtonian theory of gravity, the formula of energy conservation for the ellipse orbit is

$$\frac{1}{2}mV^2-\frac{GMm}{r}=-\frac{GMm}{2b} \quad \text{or} \quad V^2=\frac{2GM}{r}-\frac{GM}{b} \tag{126}$$

Substitute (126) in (127), we get

$$\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right)+\left(1-\rho^2\right)\frac{1}{r}=\frac{GMm^2}{K^2}\left(1-\frac{GM}{c^2 b}\right) \tag{127}$$

Here

$$\rho=\frac{\sqrt{2}GMm}{cK} \tag{128}$$

When $\rho \neq 1$, the solution of (127) is

$$r=\frac{p}{1+e\cos(\sqrt{1-\rho^2}\varphi)} \tag{129}$$

The precession vale of Mercury can be obtained from (129), but we need to know the concrete value of K. Because K is a constant, we can calculate it at any point of orbit according to (122). For the perihelion of Mercury, we have $r=r_2=4.6004\times10^{10}$ m and $b=5.7911\times10^{10}$ m as well as $V=r_2\dot{\varphi}$. The speed of Mercury at the perihelion is

$$V=\sqrt{\frac{2GM}{r_2}-\frac{GM}{b}}=5.9119\times10^4 \text{ m/s} \tag{130}$$

According to (122) and (130), we have

$$\rho=\frac{\sqrt{2}GMm}{cK}=\frac{\sqrt{2}GM\sqrt{1-V^2/c^2}}{cr_2V}$$

$$\approx\frac{\sqrt{2}GM}{cr_2V}=2.3122\times10^{-4} \tag{131}$$

So $\rho^2=5.3463\times10^{-8}<<1$, (129) can be written as

$$r=\frac{p}{1+e\cos(1-\rho^2/2)\varphi]} \tag{132}$$

Comparing with (100) and (101), the precession angle for Mercury to move around the sun for a circle is

$$\Delta\varphi=2\pi\times\frac{\rho^2}{2}=1.6796\times10^{-7} \tag{133}$$

It is 415.28 circles for the Mercury moves around the sun in a century, so the precision value is

$$\Delta\varphi'=415.28\times1.6796\times10^{-7}=6.9750\times10^{-5}=14.4'' \tag{134}$$

We see from discussion above that based on special relativity, the precession is caused by the item $\rho^2 u$ which is different from the revised item αu^2 of general relativity. According to the motion equation (119) of special relativity, the revised item caused by speed is proportion to r^{-1}, which does not cause the serious deformation of Mercury orbits.

6. Conclusion

In general relativity, the arc length is used as parameter to establish the geodesic equation. The method is improper for it will the cause self-entangled of theory and leads to serious contradiction. By using arbitrary parameter to establish geodesic equation, the motion equation of planets can be obtained based on general relativity. It is proved that general relativity can not describe the ellipse orbit. This result is fatal for the Einstein's theory of gravity of curved space-time. It is out of the equation to discuss the perihelion precession of Mercury.

By means of the accurate numerical method of computer, it is proved that the revision of general relativity would cause serious deformation of Mercury's orbits. The distances of perihelion and aphelion would be changed greatly. However, such changes have not been found in astronomic observation. So the revision of general relativity on the Newtonian theory is impossible.

The most important experiment verification of Einstein's theory of gravity is the perihelion precession of Mercury. Because the errors caused by the other planet's permutation may be far greater than 43" a century, the calculation of general relativity is actually meaningless.

On the other side, it is proved in [8] that according to general relativity strictly, the poles of light's orbits are located at the places $2.95\times10^3 \sim 2.11\times10^4$ m away from the solar center. The light omitted by stars outside the solar system would enter the sun inner and i eliminated. They can not be observed on the earth so that the night sky of the earth would be black. However, it is not true. At present, when general relativity is used to calculate the deflection of light and the delay of radar wave, the orbit poles are always chosen on the surface of the sun. The calculation results of general relativity are untrue. In four experiment verifications of general relativity, three are wrong.

Therefore, the revision of general relativity on the Newtonian theory of gravity is invalid. The revision should be carried out in flat space. For example, taking following form

$$\vec{F} = -\frac{GMm\vec{r}}{r^3} + \frac{\beta\,\vec{b}}{r^2} \qquad (135)$$

Or assume that gravity mass is related to the speed of object, the gravity is 【9】:

$$\vec{F} = -\frac{GMm\vec{r}}{r^3} f(V) \qquad (136)$$

The formula (119) is just an example.

Meanwhile, we can also introduce like-magnetism gravity to establish the theory of gravity in the similar form of electromagnetism theory. This theory is based on flat space-time without unusual or strange things which appear frequently in general relativity.

In fact, the capacity of general relativity is very limited. It is quite awkward to use general relativity to deal with practical problems. It is completely inapplicable in many situations. For example, rocket is accelerated on the earth surface to rise into space (rather than falls free in gravitational field). General relativity is incapable for such simple problem in the Newtonian mechanics.

The conclusion of this paper is that the Einstein's theory of gravity is a completely wrong one. It can not be used to replace the Newtonian theory of gravity. The development of physics is delayed for a hundred years by general relativity. We should give up the gravity theory of curved space-time and return to the description form of dynamic in flat space-time by the reformation the Newtonian theory of gravity.

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