

The Proof That There are no Invariabilities of Lorentz Transformations in the Interaction Theories of Micro-particle Physics

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Abstract It is proved in this paper that there are at least five situations in the interaction theories of micro-particle physics that the Lorentz transformations have no invariabilities. 1. In the formula to calculate transition probabilities in particle physics, the so-called invariability factor of phase space $d^3\bar{p}/E$ is not invariable actually under the Lorentz transformations. Only in the situation of one-dimensional motion with $u_y = u_z = 0$, it is invariable. 2. The polarization vectors of quantized electromagnetic fields and their summation formulas have no invariability of Lorentz transformation. 3. The propagation function of spinor field in quantum theory of field has no Lorentz invariability. In the transformation what appears is the factor $a_{\mu\nu}a_{\lambda\mu} \neq \delta_{\nu\lambda}$, rather than $a_{\mu\nu}a_{\mu\lambda} = \delta_{\nu\lambda}$, but current calculation takes $a_{\mu\nu}a_{\lambda\mu} = \delta_{\nu\lambda}$ which leads to wrong result. 4. Though the motion equations of quantum fields and the interaction Hamiltonian are unchanged under the Lorentz transformation, the motion equation of perturbation which is used to calculate the transition probability in the interaction representation has no invariability. 5. The interactions between bound state's particles have no Lorentz invariability. In fact, the principle of relativity has no approximation if it holds. 6. The calculation methods of high order perturbations normalization processes in quantum theory of fields violate the invariability of Lorentz transformation. The conclusions above are effective for strong, weak and electromagnetic interactions and so on. Therefore, the principle of relativity does not hold in the micro-particle's interactions. On the other hand, the invariability principle of light's speed is still effective. So the formulas of special relativity still hold, but we should consider them with absolute significances.

Key Words: Principle of relativity, Lorentz invariability violation, Quantum theory of field, Quantum mechanics, Phase space factor, Propagation function, Normalization, CMBR, Cosmology

1. Introduction

The Einstein's special relativity is based on two foundational principles. One is the principle of relativity and another is the invariability of light's speed. According to the principle of relativity, motion is a relative concept. We can not determine whether a reference frame is moving in a uniform speed or at rest by physical experiments. With more strict speaking, the forms of physical laws are unrelated to the choices of reference frames. To reach this aim, physical quantities should be covariant under Lorentz transformations.

Because most macro-physics involve low speed's processes, special relativity is mainly used in micro-physics processes. Micro-physics includes relativity quantum theory of field and non-relativity quantum mechanics. Because the motion equations and interactions Hamiltonians in quantum theory of field are considered invariable under Lorentz transformation, physicists believes that interaction processes of micro-particles satisfy the principle of relativity.

However, astronomic observations founded that the cosmic microwave background radiation (CMBR) was anisotropic in space distribution. If we take the reference frame in which CMBR is isotropic as an absolutely static reference frame, observations indicates that our solar system is moving in a speed about 390Km/s along a certain direction in the coordinate system of celestial sphere. This velocity can be considered as the absolute velocity of solar system. In fact, Big Bang cosmology needs an absolutely static reference frame. We can think that all initial velocities of materials in the universe were caused by the accelerating processes in Big Bang. So physics is in a dilemma situation at present. Cosmology and relativity are inconsistent. This is a serious problem.

In this paper, we prove that the principle of relativity is only a subjective and specious judgment under macroscopic and low speed's conditions, just as Galileo's intuitional experiments in a closed ship. Under microscopic and high speed's conditions, the principle of relativity does not hold. In fact, the principle of relativity has never been accurately verified by experiments. Physicists have never carried out experiments in a reference frame with high enough speed to verify the correctness of relativity principle!

In the processes of micro-particle's decays and collisions, the transition probabilities are considered having nothing to do reference frames. Quantum theory of field is constructed in this principle and the interaction theories of micro-particles are considered satisfying the principle of relativity. However, in this paper, we carefully examines the motion equations and the interaction Hamiltonians of micro-particles in quantum theory of field and finds that at least in five situations the interaction theories of micro-particles have no invariability under Lorentz transformation.

1. The so-called invariability factor of phase space d^3p/E in the formulas of transition probability has no invariability of Lorentz transformation actually. Only in the one-dimensional motion process with $u_y = u_z = 0$, it is invariable. The result indicates that all transition processes of micro-interactions have no invariabilities under Lorentz transformation. The commutation relations of field operators have no invariability due to this result.

2. The polarization vectors of quantized electromagnetic fields and their summation formulas have no invariability of Lorentz transformation, so that all micro-interaction processes related to photons have no Lorentz invariability.

3. The propagation function of spinor field has no Lorentz invariability actually. What appears in the transformation is the sum of Lorentz factors $a_{\mu\nu}a_{\lambda\mu} \neq \delta_{\nu\lambda}$ when $\nu, \lambda = 1, 4$, rather $a_{\mu\nu}a_{\mu\lambda} = \delta_{\nu\lambda}$. But in the current calculation, we take $a_{\mu\nu}a_{\lambda\mu} = \delta_{\nu\lambda}$. The confusion of subscript's position leads to serious mistake.

4. Though the motion equations of gauge fields and spinor fields and the interaction Hamiltonians are unchanged under Lorentz transformation, the motion equation of perturbation theory used to practically calculate the transition probabilities in interaction representation has no Lorentz invariability. In fact, even the most foundational normalization formulas of probability wave in quantum theory of field and quantum mechanics have no invariability of Lorentz transformation too. Unfortunately, these problems are neglected at present.

5. The interactions between bound state's particles violate the invariability of Lorentz transformation. In fact, so-called relativity quantum theory of field only describes interaction processes in which particles are free at their initial and final states. The Hamiltonians of interactions are constructed by free particle's wave functions. What are measured in experiments are free particles at final states. Because the wave functions and the products $p \cdot q$ of four-dimensional momentum of free particles are invariable quantities of Lorentz transformations, the probability amplitudes of transitions are invariable.

However, in physics, more are the interactions between bound particles in which the wave functions, energies and momentums have no symmetries of free particles. We have $p^2 \neq -m^2$ so that the product $p \cdot q$ is not the invariable quantity of Lorentz transformation again. The interaction Hamiltonians can not be constructed by free particle's wave functions and physical quantities. The method of quantum theory of field may be ineffective. In this paper, we prove that the interactions between bound state's particles have no Lorentz invariability by several examples just as the scattering process of electrons in external field, the fine structure of hydrogen atomic energy levels and the emission and absorption of photons in atoms.

According to current understanding, relativity quantum theory of field describes unstable particles with high speed, and non-relativity approximate quantum mechanics describes stable particles with low speeds. This classification is unsuitable for the principle of relativity principle. The principle of relativity has no approximation. If the principle of relativity is correct, it should also be effective for the micro-particles with low speeds. In fact, classical Newtonian theory also satisfies the principle of relativity. The motion equations of Newtonian mechanics are unchanged under the Galileo's transformation. However, the motion equations and Hamiltonians of non-relativity quantum mechanics can not keep unchanged no matter under the Galileo's transformation or the Lorentz's transformation. This fact indicates that micro-particle physics has no relativity in essence! So called non-relativities of motion equation and interaction Hamiltonians in quantum mechanics are not caused by the approximation methods of descriptions. The truth is that relativity does not exist in micro-physics at all!

6. The normalization processes of high order perturbations in quantum theory of fields violate the invariability of Lorentz transformation. We take the Lamb shift of hydrogen atomic energy levels as concrete example at first and then prove the conclusion generally.

The conclusions above are generally effective for strong, weak and electromagnetic interactions. Therefore, the principle of relativity does not hold in the fields of micro-particle's interactions. However, the invariability principle of light's speed is still effective. It means that the formulas of special relativity can still hold. But they should be explained with absolute significance.

In this way, the experiments of micro-particles and the observations of macro-cosmology become consistent and the contradiction between cosmology and special relativity can be eliminated thoroughly.

2. The Lorentz transformation of phase space factor $d^3 \vec{p} / E$

2.1 The invariable quantities of Lorentz transformation in particle physics

In particle physics, the basic formula to calculate the decay probability in particle physics is **【1】**

$$dW_{fi} = \delta^4(P - Q) \frac{K}{2^B (2\pi)^{3n-4} E_0} \sum \bar{|M_{fi}|}^2 \prod_{k=1}^n \frac{d^3 \vec{p}_k}{E_k} \quad (1)$$

The basic formula to calculate the collision cross-sections is

$$dW_{fi} = \delta^4(P - Q) \frac{K}{2^B (2\pi)^2 J} \sum \bar{|M_{fi}|}^2 \prod_{k=1}^n \frac{d^3 \vec{p}_k}{E_k} \quad (2)$$

In the formulas, the probability amplitude M_{fi} is the factor of dynamics and others are kinetic factors. $d^3 p_k / E_k$ is the factor of phase space at final state. All text books and physical documents consider this factor as the invariable quantities of Lorentz transitions. However, it is not unless in one-dimensional motion with $u_y = u_z = 0$.

Before proving it, we need to define the invariable quantity of Lorentz transformation. In special relativity, physical quantities are transformed in the forms of covariance. Suppose that there is a physical quantity $A(\bar{x}, \bar{u}, t)$ composed of space-time coordinates and velocity in the inertial reference frame K . It may be a scalar, vector or tensor (subscripts are neglected). We transform it to the reference frame K' which moves in a uniform speed V relative to K . If obtained $A'(\bar{x}', \bar{u}', V, t')$ contains relative speed V , we say that the physical quantity is not the invariable quantity of Lorentz transformation. If it does not contain V and the relation that $A'(\bar{x}', \bar{u}', t')$ depends on \bar{x}', \bar{u}', t' is the same as that $A(\bar{x}, \bar{u}, t)$ depends on \bar{x}, \bar{u}, t , we say that it is the invariable quantity of Lorentz transformation.

We take several examples commonly appearing in particle physics. Suppose that K' moves along the positive direction of x axis relative to K , the Lorentz coordinate transformations are (set $c=1$)

$$x' = \frac{x - Vt}{\sqrt{1 - V^2}} \quad y' = y \quad z' = z \quad t' = \frac{t - Vx}{\sqrt{1 - V^2}} \quad (3)$$

The inversed transformation of (3) is

$$x = \frac{x' + Vt'}{\sqrt{1 - V^2}} \quad y = y' \quad z = z' \quad t = \frac{t' + Vx'}{\sqrt{1 - V^2}} \quad (4)$$

The Lorentz transformations of velocities are

$$u_x = \frac{u'_x + V}{1 + u'_x V} \quad u_y = \frac{u'_y \sqrt{1 - V^2}}{1 + u'_x V} \quad u_z = \frac{u'_z \sqrt{1 - V^2}}{1 + u'_x V} \quad 1 - u^2 = \frac{(1 - V^2)(1 - u'^2)}{(1 + u'_x V)^2} \quad (5)$$

So t , x and velocities are not the invariable quantities of Lorentz transformation, but space coordinate y and z are unchanged. In reference frame K , momentum and energy of a particle with mass m are

$$p_x = \frac{mu_x}{\sqrt{1 - u^2}} \quad p_y = \frac{mu_y}{\sqrt{1 - u^2}} \quad p_z = \frac{mu_z}{\sqrt{1 - u^2}} \quad E = \frac{m}{\sqrt{1 - u^2}} \quad (6)$$

Substituting (5) in (6), we obtain the Lorentz transformations of momentum and energy

$$p_x = \frac{m(u'_x + V)}{\sqrt{1 - u'^2} \sqrt{1 - V^2}} \neq p'_x \quad p_y = \frac{mu'_y}{\sqrt{1 - u'^2}} = p'_y$$

$$p_z = \frac{mu'_z}{\sqrt{1 - u'^2}} = p'_z \quad E = \frac{m(1 + u'_x V)}{\sqrt{1 - u'^2} \sqrt{1 - V^2}} \neq E' \quad (7)$$

So p_y and p_z are the invariable quantities of Lorentz transformations, but p_x and E are not. Writing them in the four-dimensional forms with $p = (\bar{p}, iE)$, we have

$$p^2 = \bar{p}^2 - E^2 = \bar{p}'^2 - E'^2 = p'^2 = -m^2 \quad (8)$$

p^2 is the invariable quantities of Lorentz transformation, though it is equal to a constant. The factor J in (2) is also the Lorentz invariable quantity with

$$J = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} \quad (9)$$

We can prove it easily. According to (6), we have

$$p_1 \cdot p_2 = \frac{m_1 m_2 (u_{x1} u_{2x} - 1)}{\sqrt{1-u_1^2} \sqrt{1-u_2^2}} + p_{1y} p_{2y} + p_{1z} p_{2z} \quad (10)$$

According to (7), we get

$$\begin{aligned} p_1 \cdot p_2 &= \frac{m_1 m_2 [(u'_{1x} + V)(u'_{2x} + V) - (1 + u'_{1x} V)(1 + u'_{2x} V)]}{\sqrt{1-u_1'^2} \sqrt{1-u_2'^2} (1-V^2)} + p'_{1y} p'_{2y} + p'_{1z} p'_{2z} \\ &= \frac{m_1 m_2 (u'_{1x} u'_{2x} - 1)}{\sqrt{1-u_1'^2} \sqrt{1-u_2'^2}} + p'_{1y} p'_{2y} + p'_{1z} p'_{2z} = p'_1 \cdot p'_2 \end{aligned} \quad (11)$$

In particle physics, similar invariable quantities are

$$\begin{aligned} S &= -(p_1 + p_2)^2 = m_1^2 + m_2^2 - 2p_1 \cdot p_2 \\ t &= -(p_1 - p_2)^2 = m_1^2 + m_2^2 + 2p_1 \cdot p_2 \end{aligned} \quad (12)$$

Meanwhile, we have

$$\begin{aligned} p \cdot x &= \bar{p} \cdot \bar{x} - Et = \frac{m(u'_x + V)(x' + Vt')}{\sqrt{1-u'^2} (1-V^2)} + p'_y y' + p'_z z' - \frac{m(1 + u'_x V)(t' + Vx')}{\sqrt{1-u'^2} (1-V^2)} \\ &= \frac{mu'_x x'}{\sqrt{1-u'^2}} + p'_y y' + p'_z z' - \frac{mt'}{\sqrt{1-u'^2}} = \bar{p}' \cdot \bar{x}' - E't' = p' \cdot x' \end{aligned} \quad (13)$$

So the wave function of free particle is also the invariable quantities of Lorentz transformation with

$$\psi(\bar{x}, t) = Ae^{i(\bar{p} \cdot \bar{x} - Et)} \rightarrow Ae^{i(\bar{p}' \cdot \bar{x}' - E't')} = \psi'(\bar{x}', t') \quad (14)$$

The definition of δ function in particle physics is

$$\delta^4(p) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} e^{ipx} d^4x \quad (15)$$

Transforming it to K' reference frame, because dy and dz are unchanged, we only need to calculate the relation between $dxdt$ and $dx'dt'$. By using the Jacobi's matrix of integral transformation, we have

$$dxdt = \begin{vmatrix} \partial x / \partial x' & \partial x / \partial t' \\ \partial t / \partial x' & \partial t / \partial t' \end{vmatrix} dx'dt' = \begin{vmatrix} 1/\sqrt{1-V^2} & V/\sqrt{1-V^2} \\ V/\sqrt{1-V^2} & 1/\sqrt{1-V^2} \end{vmatrix} dx'dt' = dx'dt' \quad (16)$$

So d^4x and (16) are also the invariable quantity of Lorentz transformation with $\delta^4(p) = \delta^4(p')$.

2.2 The Lorentz transformation of phase space factor

If phase space factor is the invariable quantity of Lorentz transformation, it should satisfy

$$\frac{d^3 p}{E} = \frac{dp_x dp_y dp_z}{E} = \frac{dp'_x dp'_y dp'_z}{E'} = \frac{d^3 p'}{E'} \quad (17)$$

We will prove that (17) only holds in the situation of one-dimensional motion with $u_y = u_z = 0$. In general situation when $u_y \neq 0$ and $u_z \neq 0$, (17) is untenable.

According to (6), we have

$$\frac{dp_x}{E} = \sqrt{1-u^2} \left[\frac{du_x}{\sqrt{1-u^2}} + u_x d \frac{1}{\sqrt{1-u^2}} \right] = du_x + \frac{u_x du^2}{2(1-u^2)} \quad (18)$$

For the one-dimensional motion with $u^2 = u_x^2$, $du^2 = 2u_x du_x$, (18) can be written as

$$\frac{dp}{E} = \frac{du}{1-u^2} \quad (19)$$

By transform to K' reference frame, according to (5), we have

$$du_x = \frac{(1-V^2)}{(1+u'_x V)^2} du'_x \quad 1-u^2 = \frac{(1-V^2)(1-u'^2)}{(1+u'_x V)^2} \quad (20)$$

Substituting (20) in (9), we get

$$\frac{dp_x}{E} = \frac{du_x}{1-u^2} = \frac{du'_x}{1-u'^2} = \frac{dp'_x}{E'} \quad (21)$$

Therefore, it is proved that under the situation of one-dimensional motion, the phase space factor can not be invariable under the Lorentz transformation.

However, the current quantum theory of field generalized this result to the situation of three-dimensional motion without proof. It is proved below that this is impossible.

In the three-dimensional motion with $u_y \neq 0$, $u_z \neq 0$, $du^2 = 2(u_x du_x + u_y du_y + u_z du_z)$, according to (5) and (18), we have

$$\begin{aligned} \frac{dp_x}{E} &= du_x + \frac{u_x(u_x du_x + u_y du_y + u_z du_z)}{(1-u^2)} = \frac{(1-V^2)}{(1+u'_x V)^2} du'_x \\ &+ \frac{(u'_x + V)}{2(1-u'^2)(1+u'_x V)^2} \left[(1+u'_x V) du'^2 + 2(1-u'^2) V du' \right] \\ &= \frac{(1-V^2)}{(1+u'_x V)^2} du'_x + \frac{(u'_x + V)}{(1+u'_x V)^2} V du'_x + \frac{(u'_x + V) du'^2}{2(1-u'^2)(1+u'_x V)} \\ &= \frac{1}{(1+u'_x V)} \left[du'_x + \frac{u'_x du'^2}{2(1-u'^2)} + \frac{V du'^2}{2(1-u'^2)} \right] \neq \frac{du'_x}{1-u'^2} \end{aligned} \quad (22)$$

Comparing with (21), it is obvious that (22) can not keep unchanged under, the Lorentz transformation. The result can be verified directly. For the one-dimensional motion with $u'^2 = u_x'^2$, $du'^2 = 2u'_x du'_x$, we substitute the relations in the right side of (22) and get (21) immediately.

Then according to (7), i.e., $p_y = p'_y$, $p_z = p'_z$, $dp_y = dp'_y$, $dp_z = dp'_z$, we get

$$\frac{dp_x}{E} \neq \frac{dp'_x}{E'} \quad \text{or} \quad \frac{dp_x dp_y dp_z}{E} \neq \frac{dp'_x dp'_y dp'_z}{E'} \quad (23)$$

For the concrete calculation, by considering (7), we have

$$dp_x \rightarrow d\left(\frac{m(u'_x + V)}{\sqrt{1-u'^2}\sqrt{1-V^2}}\right) = \frac{dp'_x}{\sqrt{1-V^2}} + \frac{mV}{\sqrt{1-V^2}} d\frac{1}{\sqrt{1-u'^2}}$$

$$E \rightarrow E'_L = \frac{E' + mVu'_x}{\sqrt{1-V^2}} = \frac{E'}{\sqrt{1-V^2}} \left(1 + \frac{mVp'_x}{E'}\right) \quad (24)$$

$$d\frac{mu'_x}{\sqrt{1-u'^2}} = mu'_x d\frac{1}{\sqrt{1-u'^2}} + \frac{m}{\sqrt{1-u'^2}} du'_x = \frac{p'_x}{E'} d\frac{1}{\sqrt{1-u'^2}} + \frac{m}{\sqrt{1-u'^2}} du'_x \quad (25)$$

We obtain

$$\frac{dp_x dp_y dp_z}{E} = \frac{1 + VE'/p'_x}{E'(1 + mVp'_x/E')} dp'_x dp'_y dp'_z - \frac{V}{u'_x(1 + mVp'_x/E')} du'_x dp'_y dp'_z \quad (26)$$

By using the Jacobi's formula, we get

$$dp'_y dp'_z = \begin{vmatrix} \partial p'_y / \partial u'_y & \partial p'_y / \partial u'_z \\ \partial p'_z / \partial u'_y & \partial p'_z / \partial u'_z \end{vmatrix} du'_y du'_z = \frac{m^2(1-u'^2)}{(1-u'^2)^2} du'_y du'_z \quad (27)$$

Here $u'^2 = u'^2_x + u'^2_y + u'^2_z$. Suppose that the integrand function is unchanged with $f(p) = f(p')$, we get

$$\int_{-\infty}^{\infty} \frac{f(p)}{E} dp_x dp_y dp_z = \int_{-\infty}^{\infty} \frac{f(p')(1 + VE'/p'_x)}{E'(1 + mVp'_x/E')} dp'_x dp'_y dp'_z$$

$$- \int_{-c}^c \frac{f(p')V}{u'_x(1 + mVu'_x/E')} \frac{m^2(1-u'^2)}{(1-u'^2)^2} du'_x du'_y du'_z \neq \int_{-\infty}^{\infty} \frac{f(p')}{E'} dp'_x dp'_y dp'_z \quad (28)$$

The up and down limits of integral signals about $du'_x du'_y du'_z$ are $\pm c$. The reason is that when $p'_i \rightarrow \pm\infty$, we have $u'_i \rightarrow \pm c$.

3. The Lorentz transformations of the motion equation of spinor field and the Hamiltonian of electromagnetic interaction

3.1 The Lorentz transformation matrix of four-dimensional space-time coordinates

In special relativity, we take $x = (x_1, x_2, x_3, x_4) = (x, y, z, it)$ and write (3) as $x'_\mu = a_{\mu\nu} x_\nu$ with

$$a_{\mu\nu} = \begin{pmatrix} \gamma & 0 & 0 & iV\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -iV\gamma & 0 & 0 & \gamma \end{pmatrix} \quad (29)$$

Here $\gamma = 1/\sqrt{1-V^2}$. The inversed transformation (4) is $x_\mu = a_{\nu\mu} x'_\nu$. According to the invariability principle of light's speed, we have $x'_\mu x'_\mu = a_{\mu\nu} a_{\mu\lambda} x_\nu x_\lambda = x_\nu x_\nu$, so we have $a_{\mu\nu} a_{\mu\lambda} = \delta_{\nu\lambda}$ **[2]**. We should pay attention to the positions of indexes. According to (29), we have $a_{\mu\nu} a_{\lambda\mu} \neq \delta_{\nu\lambda}$ or $a_{\nu\mu} a_{\mu\lambda} \neq \delta_{\nu\lambda}$ when $\nu, \lambda = 1, 4$. By writing them out clearly, we have $a_{2\mu} a_{\mu 2} = 1$, $a_{3\mu} a_{\mu 3} = 1$ and

$$a_{1\mu} a_{\mu 1} = a_{4\mu} a_{\mu 4} = \gamma^2 + (iV\gamma)(-iV\gamma) = \frac{1+V^2}{1-V^2} \neq 1 \quad (30)$$

The others are zero. This result is very significant for our discussions below. It is proved that what appears in the Lorentz transformation of propagation functions of spinor fields is $a_{\nu\mu}a_{\mu\lambda} \neq \delta_{\nu\lambda}$ when $\nu, \lambda = 1, 4$, rather than $a_{\mu\nu}a_{\mu\lambda} = \delta_{\nu\lambda}$. But in the current calculation, we take $a_{\mu\nu}a_{\lambda\mu} = \delta_{\nu\lambda}$. The confusion of subscript's position leads to wrong result, so that we think that propagation functions of spinor fields is Lorentz invariable.

3.2 The Lorentz transformations of spinor field and its motion equation

The Dirac equation of free spinor field in K reference frame is

$$(\gamma_\mu \partial_\mu + m)\psi(x) = 0 \quad (31)$$

By transforming it to K' reference frame and considering $x'_\nu = a_{\nu\mu}x_\mu$, we have following differential relations 【2】

$$\frac{\partial}{\partial x_\mu} = \frac{\partial x'_\nu}{\partial x_\mu} \frac{\partial}{\partial x'_\nu} = a_{\nu\mu} \frac{\partial}{\partial x'_\nu} \quad \text{or} \quad \partial_\mu = a_{\nu\mu} \partial'_\nu \quad (32)$$

The covariant rule of differential operator ∂_μ is consistent with that of coordinate transformation $x_\mu = a_{\nu\mu}x'_\nu$. We use subscript L representing direct Lorentz transformation, $\psi_L(x', V)$ representing the direct Lorentz transformations of coordinates in function $\psi(x)$. Substituting (32) in (31) and let $x = a^{-1}x'$ represent $x_\mu = a_{\nu\mu}x'_\nu$, we get

$$(a_{\nu\mu} \gamma_\mu \partial'_\nu + m)\psi(a^{-1}x') = (a_{\nu\mu} \gamma_\mu \partial'_\nu + m)\psi_L(x', V) = 0 \quad (33)$$

By introducing matrix Λ and acting it on the left side of (33), we get

$$\Lambda(a_{\nu\mu} \gamma_\mu \partial'_\nu + m)\psi_L(x', V) = (a_{\nu\mu} \Lambda \gamma_\mu \Lambda^{-1} \partial'_\nu + m)\Lambda \psi_L(x', V) = 0 \quad (34)$$

Suppose that there exists reversal matrix Λ^{-1} and let

$$a_{\nu\mu} \Lambda \gamma_\mu \Lambda^{-1} = \gamma_\nu \quad \text{or} \quad \Lambda^{-1} \gamma_\mu \Lambda = a_{\mu\nu} \gamma_\nu \quad (35)$$

It has been proved $\Lambda = \cosh(\theta/2) + i\gamma_1\gamma_4 \sinh(\theta/2)$ with $\cosh\theta = \gamma$ and $\sinh\theta = V\gamma$ in quantum theory of field 【2】. It can be written as the form of matrix

$$\Lambda = \begin{pmatrix} \cosh \theta/2 & 0 & 0 & -\sinh \theta/2 \\ 0 & \cosh \theta/2 & -\sinh \theta/2 & 0 \\ 0 & -\sinh \theta/2 & \cosh \theta/2 & 0 \\ -\sinh \theta/2 & 0 & 0 & \cosh \theta/2 \end{pmatrix} \quad (36)$$

Defining new wave function $\psi'(x')$ with

$$\psi'(x', V) = \Lambda \psi(x) = \psi_L(x', L) \quad (37)$$

or

$$\psi(x) = \psi_L(a^{-1}x', V) = \Lambda^{-1} \psi'(x', L) \quad (38)$$

(31) can be written as

$$(\gamma_\nu \partial'_\nu + m)\psi'(x', V) = 0 \quad (39)$$

(39) and (31) have the same form, so we say that the motion equation of free spinor field is invariable under the Lorentz transformation. However, $\psi'(x', V)$ contains the speed of relative motion. The difference

exists practically. The current quantum theory of field neglected this difference and lead to the invariability of the Lorentz transformation.

On the other hand, it was proved $\gamma_4 \Lambda^+ \gamma_4 = \Lambda^{-1}$ in quantum theory of field [1]. Based on it, we get $\Lambda = \gamma_4 (\Lambda^{-1})^+ \gamma_4$. By considering relation $\gamma_4^2 = 1$ and (36), we obtain

$$\bar{\psi}'(x) = \psi'^+(x', V) (\Lambda^{-1})^+ \gamma_4 = \psi'^+(x', V) \gamma_4 \gamma_4 (\Lambda^{-1})^+ \gamma_4 = \bar{\psi}'(x', V) \Lambda \quad (40)$$

Or
$$\bar{\psi}'(x', V) = \bar{\psi}'(x) \Lambda^{-1} = \bar{\psi}'_L(x', V) \Lambda^{-1} \quad (41)$$

3.3 The Lorentz transformation of the Hamiltonian of electromagnetic interaction

In quantum theory of field, by considering the interaction between electromagnetic field and spinor field, the motion equation in K reference frame is

$$\gamma_\mu (\partial_\mu - ie A_\mu) \psi + m \psi = 0 \quad (\partial_\mu + ie A_\mu) \bar{\psi} \gamma_\mu - m \bar{\psi} = 0 \quad (42)$$

$$\partial^2 A_\mu = -J_\mu \quad J_\mu = \frac{ie}{2} (\bar{\psi} \gamma_\mu \psi - \psi^T \gamma_\mu^T \bar{\psi}^T) \quad \mathcal{H}(x) = -A_\mu J_\mu \quad (43)$$

Here $\mathcal{H}(x)$ is the Hamiltonian of interaction. According to the covariant rule of vector in special relativity, similar to the coordinate transformation $x_\mu = a_{\nu\mu} x'_\nu$, the covariant formula of four-dimensional electromagnetic potential $A_\mu(x) = (\bar{A}, i\varphi)$ is defined **【3】**:

$$A_\mu(x) = \frac{\partial x'_\nu}{\partial x_\mu} A_{\nu L}(x', V) = a_{\nu\mu} A'_\nu(x', V) \quad (44)$$

We write out the concrete results

$$\begin{aligned} A_x(x) &= \frac{A'_x(x', V) + V\varphi'(x', V)}{\sqrt{1-V^2}} & A_y(x) &= A'_y(x', V) \\ A_z(x) &= A'_z(x', V) & \varphi(x) &= \frac{\varphi'(x', V) + VA'_x(x', V)}{\sqrt{1-V^2}} \end{aligned} \quad (45)$$

The motion equation (42) of non-free spinor particle contains the item $\gamma_\mu A_\mu \psi$. By producing Λ it on the left side and considering (35), (37) and (41), we get

$$\Lambda A_\mu \gamma_\mu \psi = a_{\nu\mu} A'_\nu(x', V) \Lambda \gamma_\mu \Lambda^{-1} \psi'(x', V) = A'_\nu(x', V) \gamma_\nu \psi'(x', V) \quad (46)$$

So the motion equation of non-free spinor particle seems to be Lorentz invariable superficially, but the result contains the relative speed. Similarly we have

$$\begin{aligned} A_\mu(x) \bar{\psi} \gamma_\mu \psi(x) &= a_{\nu\mu} A'_\nu(x', V) \bar{\psi}'(x', V) \Lambda \gamma_\mu \Lambda^{-1} \psi'(x', V) \\ &= A'_\nu(x', V) \bar{\psi}'(x', V) \gamma_\nu \psi'(x', V) \end{aligned} \quad (47)$$

Because relative speed is contained in the transformed wave functions, the interaction Hamiltonian is unchanged under the Lorentz transformation. Just because quantum theory of field uses the functions of fee

particles to construct the Hamiltonian in the interaction representation, deals with the problems of unbounded states. As shown in (14), the functions of free particles are unchanged, so we can consider (47) as invariable.

However, as proved below, in the quantized functions of free electromagnetic fields, pole vectors and the formulas of summation are not invariable. If the problems of bounded states are involved, relative speeds can not be eliminated in general in the interaction Hamiltonian. In these cases, there are no Lorentz invariability.

3.4 The Lorentz transformation of basic equation of perturbation and the Normalization Formula of Probability Wave

However, in quantum theory of field, we calculate particle's transition probabilities in interaction representation. The basic equation of perturbation used practically is **【2】**

$$i \frac{\partial}{\partial t} |t\rangle = \mathcal{H} |t\rangle \quad (48)$$

In fact, the probability amplitude M_{fi} in (1) and (2) are deduced based on (47). It is easy to see that (47) has no invariability even though the interaction Hamiltonian is unchanged under the Lorentz transformation, the reason is that operator $\partial / \partial t$ can not keep unchanged. Let $|t'\rangle_L$ represent the Lorentz transformation of $|t\rangle$, according to (32), we have the Lorentz transformation of (48) is

$$\frac{1}{\sqrt{1-V^2}} \left[\frac{\partial}{\partial t'} + V \frac{\partial}{\partial x'} \right] |t'\rangle_L = \mathcal{H} |t'\rangle_L \quad (49)$$

Using (49) to do calculation, the transition probability is certainly different from that based on (47). Unfortunately, this problem is neglected in quantum theory of field at present.

4. The Lorentz transformations of polarization vectors of quantized electromagnetic fields and summation formulas

In quantum theory of field, the quantized free electromagnetic field is represented

$$A_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\vec{k}}{\sqrt{2\omega}} \sum_{\sigma=1}^4 \varepsilon_\mu^\sigma(k) \left[a_\sigma(k) e^{ikx} + a_\sigma^\dagger(k) e^{-ikx} \right] \quad (50)$$

In the formula, $\varepsilon_\mu^\sigma(k)$ are the four-dimensional polarization vectors. Suppose that the electromagnetic wave propagates along the direction \vec{k} , the definitions of polarization vectors are **【2】**:

$$\varepsilon_\mu^1(k) = \left[\vec{n}^1, 0 \right] \quad \varepsilon_\mu^2(k) = \left[\vec{n}^2, 0 \right] \quad \varepsilon_\mu^3(k) = \left[\vec{k} / |\vec{k}|, 0 \right] \quad \varepsilon_\mu^4(k) = \left[0, 1 \right] \quad (51)$$

Here $\vec{n}^1 = \vec{r}_1 / |r_1|$ and $\vec{n}^2 = \vec{r}_2 / |r_2|$ are orthogonal each other with

$$\vec{n}^1 \cdot \vec{n}^2 = 0 \quad \vec{k} \cdot \vec{n}^1 = 0 \quad \vec{k} \cdot \vec{n}^2 = 0 \quad (52)$$

Based on (51) and (52), there are following summation relations for polarization vectors.

$$\sum_{\mu=1}^4 \varepsilon_{\mu}^{\sigma} \varepsilon_{\mu}^{\tau} = \delta_{\sigma\tau} \quad \sum_{\sigma=1}^4 \varepsilon_{\mu}^{\sigma} \varepsilon_{\nu}^{\sigma} = \delta_{\mu\nu} \quad (53)$$

(53) is used in the calculations of transition probabilities between photons and other particles.

However, it is to prove that (53) has no invariability of Lorentz transformation. The Lorentz transformations of photon's four-dimensional momentum are **[4]**:

$$k_x = \frac{k'_x + k'V}{\sqrt{1-V^2}} \quad k_y = k'_y \quad k_z = k'_z \quad k = \frac{k' + k'_x V}{\sqrt{1-V^2}} \quad (54)$$

By considering (4) and (54), we have

$$\begin{aligned} \bar{n}^1 \cdot \bar{n}^2 &= \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \sqrt{x_2^2 + y_2^2 + z_2^2}} = \frac{\frac{1}{1-V^2} (x'_1 + Vt)(x'_2 + Vt) + y'_1 y'_2 + z'_1 z'_2}{\sqrt{\frac{(x'_1 + Vt)^2}{1-V^2} + y_1^2 + z_1^2} \sqrt{\frac{(x'_2 + Vt)^2}{1-V^2} + y_2^2 + z_2^2}} \\ &\neq \frac{x'_1 x'_2 + y'_1 y'_2 + z'_1 z'_2}{\sqrt{x_1'^2 + y_1'^2 + z_1'^2} \sqrt{x_2'^2 + y_2'^2 + z_2'^2}} = \bar{n}'^1 \cdot \bar{n}'^2 \end{aligned} \quad (55)$$

$$\begin{aligned} \bar{k} \cdot \bar{n}^1 &= \frac{k_x x_1 + k_y y_1 + k_z z_1}{\sqrt{k_x^2 + k_y^2 + k_z^2} \sqrt{x_1^2 + y_1^2 + z_1^2}} = \frac{\frac{1}{1-V^2} (k'_x + k'V)(x'_1 + Vt) + k'_y y'_1 + k'_z z'_1}{\sqrt{\frac{(k'_x + k'V)^2}{1-V^2} + k_y'^2 + k_z'^2} \sqrt{\frac{(x'_1 + Vt)^2}{1-V^2} + y_1^2 + z_1^2}} \\ &\neq \frac{k'_x x'_1 + k'_y y'_1 + k'_z z'_1}{\sqrt{k_x'^2 + k_y'^2 + k_z'^2} \sqrt{x_1'^2 + y_1'^2 + z_1'^2}} = \bar{k}' \cdot \bar{n}'^1 \end{aligned} \quad (56)$$

There are the Lorentz transformations

$$\sum_{\mu=1}^4 \varepsilon_{\mu}^1 \varepsilon_{\mu}^2 = \varepsilon_1^1 \varepsilon_1^2 + \varepsilon_2^1 \varepsilon_2^2 + \varepsilon_3^1 \varepsilon_3^2 + \varepsilon_4^1 \varepsilon_4^2 = \bar{n}^1 \cdot \bar{n}^2 + 0 \neq \bar{n}'^1 \cdot \bar{n}'^2 + 0 = \sum_{\mu=1}^4 \varepsilon_{\mu}^{\prime 1} \varepsilon_{\mu}^{\prime 2} \quad (57)$$

$$\sum_{\mu=1}^4 \varepsilon_{\mu}^1 \varepsilon_{\mu}^3 = \varepsilon_1^1 \varepsilon_1^3 + \varepsilon_2^1 \varepsilon_2^3 + \varepsilon_3^1 \varepsilon_3^3 + \varepsilon_4^1 \varepsilon_4^3 = \bar{n}^1 \cdot \bar{k} + 0 \neq \bar{n}'^1 \cdot \bar{k}' + 0 = \sum_{\mu=1}^4 \varepsilon_{\mu}^{\prime 1} \varepsilon_{\mu}^{\prime 3} \quad (58)$$

$$\begin{aligned} \sum_{\sigma=1}^4 \varepsilon_1^{\sigma} \varepsilon_2^{\sigma} &= \varepsilon_1^1 \varepsilon_2^1 + \varepsilon_1^2 \varepsilon_2^2 + \varepsilon_1^3 \varepsilon_2^3 + \varepsilon_1^4 \varepsilon_2^4 = n_x^1 \cdot n_y^1 + n_x^2 \cdot n_y^2 + k_x \cdot k_y / |\bar{k}|^2 + 0 \\ &= \frac{x_1 y_1}{|\bar{r}_1|^2} + \frac{x_2 y_2}{|\bar{r}_2|^2} + \frac{k_x k_y}{|\bar{k}|^2} = \frac{(x'_1 + Vt) y'_1}{\sqrt{1-V^2} \left(\frac{(x'_1 + Vt)^2}{1-V^2} + y_1'^2 + z_1'^2 \right)} \\ &+ \frac{(x'_2 + Vt) y'_2}{\sqrt{1-V^2} \left(\frac{(x'_2 + Vt)^2}{1-V^2} + y_2'^2 + z_2'^2 \right)} + \frac{(k'_x + k'V) k'_y}{\sqrt{1-V^2} \left(\frac{(k'_x + k'V)^2}{1-V^2} + k_y'^2 + k_z'^2 \right)} \end{aligned}$$

$$\neq \frac{x'_1 y'_1}{|\vec{r}'_1|^2} + \frac{x'_2 y'_2}{|\vec{r}'_2|^2} + \frac{k'_x k'_y}{|\vec{k}'|^2} + 0 = \varepsilon_1'^1 \varepsilon_2'^1 + \varepsilon_1'^2 \varepsilon_2'^2 + \varepsilon_1'^3 \varepsilon_2'^3 + \varepsilon_1'^4 \varepsilon_2'^4 = \sum_{\sigma=1}^4 \varepsilon_1'^{\sigma} \varepsilon_2'^{\sigma} \quad (59)$$

And so on. (53) can not keep unchanged under the transformations. As shown below, the corresponding transition probabilities have no invariability.

5. The Lorentz transformations of commutation relations of field operators and propagation functions

5.1 The Lorentz transformations of commutation relations and propagation functions of scalar fields

In quantum theory of field, the operators of scalar fields can be written as:

$$\begin{aligned} \varphi(x) &= \varphi^{(-)}(x) + \varphi^{(+)}(x) & \varphi^+(x) &= \varphi^{+(-)}(x) + \varphi^{+(+)}(x) & (60) \\ \varphi^{(-)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \frac{d^3k}{\sqrt{2\omega}} a(k) e^{ikx} & \varphi^{(+)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \frac{d^3k}{\sqrt{2\omega}} a^+(k) e^{ikx} \\ \varphi^{+(-)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \frac{d^3k}{\sqrt{2\omega}} b(k) e^{ikx} & \varphi^{+(+)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \frac{d^3k}{\sqrt{2\omega}} b^+(k) e^{ikx} & (61) \end{aligned}$$

Their commutation relations are

$$i\Delta^{(-)}(x_1 - x_2) = [\varphi^{(+)}(x_1), \varphi^{+(-)}(x_2)] = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d^3k}{2\omega} e^{-ik(x_1 - x_2)} \quad (62)$$

$$i\Delta^{(+)}(x_1 - x_2) = [\varphi^{(-)}(x_1), \varphi^{+(+)}(x_2)] = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d^3k}{2\omega} e^{ik(x_1 - x_2)} \quad (63)$$

As mentioned before, d^3k/ω is not Lorentz invariable quantity, so the commutation relations of scalar fields have no Lorentz symmetries. The propagation function of scalar field is

$$\begin{aligned} \Delta_F(x_1 - x_2) &= \theta(t_1 - t_2) i\Delta^{(+)}(x_1 - x_2) - \theta(t_2 - t_1) i\Delta^{(-)}(x_1 - x_2) \\ &= -\frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} \frac{d^4k}{k^2 + m^2} e^{ik(x_1 - x_2)} \quad (64) \end{aligned}$$

According to (17), d^4k is invariable under Lorentz transformation. The other quantities in (64) are Lorentz invariable. So the propagation functions of scalar fields are unchanged under the Lorentz transformation. The situations of electromagnetic field are similar to scalar fields (with $m \rightarrow 0$). The commutation relations have no symmetries but the propagation functions have under Lorentz transformations.

5.2 The Lorentz transformations of commutation relations and the propagation functions of spinor fields

In quantum theory of field, the operators of spinor fields can be written as

$$\psi(x) = \psi^{(-)}(x) + \psi^{(+)}(x) \quad \bar{\psi}(x) = \bar{\psi}^{(-)}(x) + \bar{\psi}^{(+)}(x) \quad (65)$$

$$\begin{aligned} \psi^{(-)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \sqrt{\frac{m}{E}} \sum_{r=1}^2 u_r(p) b_r(p) e^{ipx} d^3 p \\ \psi^{(+)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \sqrt{\frac{m}{E}} \sum_{r=1}^2 v_r(p) d_r^+(p) e^{-ipx} d^3 p \\ \bar{\psi}^{(-)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \sqrt{\frac{m}{E}} \sum_{r=1}^2 \bar{v}_r(p) d_r(p) e^{ipx} d^3 p \\ \bar{\psi}^{(+)}(x) &= \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} \sqrt{\frac{m}{E}} \sum_{r=1}^2 \bar{u}_r(p) b_r^+(p) e^{-ipx} d^3 p \end{aligned} \quad (66)$$

The commutation relations are

$$\{\psi_{\alpha}^{(-)}(x_1), \bar{\psi}_{\beta}^{(+)}(x_2)\} = i(-\gamma_{\mu} \partial_{\mu} + m)_{\alpha\beta} \Delta^{(+)}(x_1 - x_2) \quad (67)$$

$$\{\psi_{\alpha}^{(+)}(x_1), \bar{\psi}_{\beta}^{(-)}(x_2)\} = i(-\gamma_{\mu} \partial_{\mu} + m)_{\alpha\beta} \Delta^{(-)}(x_1 - x_2) \quad (68)$$

Here

$$\Delta^{(-)}(x_1 - x_2) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \frac{d^3 p}{2E} e^{-ip(x_1 - x_2)} \quad \Delta^{(+)}(x_1 - x_2) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \frac{d^3 p}{2E} e^{ip(x_1 - x_2)} \quad (69)$$

Similarly, because $d^3 \bar{p}/E$ is not the invariable quantity of Lorentz transformation, let $\Delta_L'^{(+)}(x'_1 - x'_2)$ is the direct Lorentz transformation of space-time coordinate of $\Delta^{(+)}(x_1 - x_2)$, we have

$$\Delta_L'^{(-)}(x'_1 - x'_2) \neq \Delta'^{(-)}(x'_1 - x'_2) \quad \Delta_L'^{(+)}(x'_1 - x'_2) \neq \Delta'^{(+)}(x'_1 - x'_2) \quad (70)$$

By transforming to the motion reference frame, according to (38) and (40), we have $\psi_{\alpha}^{(-)}(x) = \Lambda_{\alpha\sigma}^{-1} \psi_{\sigma}^{\prime(-)}(x', V)$ and $\bar{\psi}_{\beta}^{(+)}(x) = \bar{\psi}_{\rho}^{\prime(+)}(x', V) \Lambda_{\rho\beta}$. By using $\{\psi_{\alpha}^{\prime(-)}(x'_1, V), \bar{\psi}_{\beta}^{\prime(+)}(x'_2, V)\}_L$ to represent the direct Lorentz transformation of (67) and considering (32), we get

$$\begin{aligned} \{\psi_{\alpha}^{(-)}(x_1), \bar{\psi}_{\beta}^{(+)}(x_2)\} &= \Lambda_{\alpha\sigma}^{-1} \{\psi_{\sigma}^{\prime(-)}(x'_1, V), \bar{\psi}_{\rho}^{\prime(+)}(x'_2, V)\}_L \Lambda_{\rho\beta} \\ &= \Lambda_{\alpha\sigma}^{-1} [i(-\gamma_{\mu})_{\sigma\rho} a_{\nu\mu} \partial'_\nu + m \delta_{\sigma\rho}] \Lambda_{\rho\beta} \Delta_L'^{(+)}(x'_1 - x'_2) \end{aligned} \quad (71)$$

According to (35) and considering $a_{\nu\mu} a_{\mu\lambda} \neq \delta_{\nu\lambda}$ when $(\nu, \lambda) = (1, 4)$, we have

$$a_{\nu\mu} \Lambda_{\alpha\sigma}^{-1} (\gamma_{\mu})_{\sigma\rho} \Lambda_{\rho\beta} \partial'_\nu = a_{\nu\mu} a_{\mu\lambda} (\gamma_{\lambda})_{\alpha\beta} \partial'_\nu \neq (\gamma_{\nu})_{\alpha\beta} \partial'_\nu \quad (72)$$

Considering $\Delta_L'^{(-)}(x'_1 - x'_2) \neq \Delta'^{(-)}(x'_1 - x'_2)$, we have

$$\{\psi_{\alpha}^{(-)}(x_1), \bar{\psi}_{\beta}^{(+)}(x_2)\} \neq i(-\gamma_{\nu} \partial'_\nu + m)_{\alpha\beta} \Delta'^{(+)}(x'_1 - x'_2) \quad (73)$$

There are mistake in the calculation of this problem in current quantum theory of field. Please see appendix for detail. Similarly, the transformation of (68) is

$$\{\psi_{\alpha}^{(+)}(x_1), \bar{\psi}_{\beta}^{(-)}(x_2)\} \neq i(-\gamma_{\nu} \partial'_\nu + m)_{\alpha\beta} \Delta'^{(-)}(x'_1 - x'_2) \quad (74)$$

Therefore, the commutation relations of spinor fields have no Lorentz symmetry too. The propagation function of spinor field is defined as

$$\begin{aligned} S_F(x_1 - x_2)_{\alpha\beta} &= \theta(t_1 - t_2) \{ \psi_{\alpha}^{(-)}(x_1), \bar{\psi}_{\beta}^{(+)}(x_2) \} - \theta(t_2 - t_1) \{ \psi_{\alpha}^{(+)}(x_1), \bar{\psi}_{\beta}^{(-)}(x_2) \} \\ &= (-\gamma_{\mu} \partial_{\mu} + m)_{\alpha\beta} \Delta_F(x_1 - x_2) = -\frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} d^4 p \frac{(m - i r_{\mu} p_{\mu} e^{ip(x_1 - x_2)})}{p^2 + m^2} \end{aligned} \quad (75)$$

According to (72) and (74), the second equal sign in (76) does not hold by the Lorentz transformation. We have

$$S_F(x_1 - x_2)_{\alpha\beta} \neq (-\gamma_{\mu} \partial'_{\mu} + m)_{\alpha\beta} \Delta'_F(x_1 - x_2) = S'_F(x'_1 - x'_2)_{\alpha\beta} \quad (76)$$

The propagation function of spinor field has no invariability under the Lorentz transformation. Let

$$\Delta_F(x_1 - x_2) = -\frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} d^4 p \frac{e^{ip(x_1 - x_2)}}{p^2 + m^2} \quad (77)$$

It is easy to know that (77) is unchanged under the Lorentz transformation. We have

$$\Delta_F(x_1 - x_2) = \Delta'_{FL}(x_1 - x_2) = -\frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} d^4 p' \frac{e^{ip'(x'_1 - x'_2)}}{p'^2 + m^2} \quad (78)$$

The Lorentz transformation of the propagation function of spinor field is

$$\begin{aligned} S_F(x_1 - x_2)_{\alpha\beta} &\rightarrow (-a_{\nu\mu} a_{\mu\lambda} \gamma_{\lambda} \partial'_{\nu} + m)_{\alpha\beta} \Delta'_{FL}(x'_1 - x'_2) \\ &= -\frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} d^4 p' \frac{(m - i a_{\nu\mu} a_{\mu\lambda} \gamma_{\lambda} p'_{\nu}) e^{ip'(x'_1 - x'_2)}}{p'^2 + m^2} \\ &\neq -\frac{i}{(2\pi)^4} \int_{-\infty}^{\infty} d^4 p' \frac{(m - i \gamma_{\nu} p'_{\nu}) e^{ip'(x'_1 - x'_2)}}{p'^2 + m^2} \end{aligned} \quad (79)$$

It is not an invariability of Lorentz transformation.

6. The Lorentz transformations of probability amplitudes in the low order processes of quantum theory of field

6.1 The Lorentz transformation of first order decay processes

The first order process describes particle's decay in quantum theory of field. In the formula (1), E_0 is particle's energy. Because E_0 is not invariable quantity under Lorentz transformation, (1) is not invariable too. In fact, the relation between particle's life and decay probability is $\tau_{fi} \sim W_{fi}^{-1}$. According to special relativity, time is depends on reference frame. In the reference frame in which particle is at rest, particle's life is longest. Only by this fact, we can say that the decay processes of micro-particles have no symmetry of Lorentz transformation. In addition, $d^3 \vec{p} / E$ is not invariable quantity which also leads to the symmetry violation of Lorentz transformation in the first order decay process.

6.2 The Lorentz transformation of second order collision processes

The transition probability of second order collision process is described by (2). Because the propagation function of spinor field has no invariability of Lorentz transformation, the probability amplitude M_{fi} containing the propagation lines of Fermion violates the symmetry of Lorentz transformation. For example, for the Compton scattering process

$$\begin{array}{ccccccc} \gamma & + & e & \rightarrow & \gamma & + & e \\ (\vec{k}, iE_k) & & (\vec{p}, iE_p) & & (\vec{l}, iE_l) & & (\vec{q}, iE_q) \end{array}$$

the transition probability amplitude is **【2】**

$$M_{fi} = \bar{u}_s(q) \left(\hat{\varepsilon}^\tau(l) \frac{i(\hat{p} + \hat{k}) - m}{(p+k)^2 + m^2} \hat{\varepsilon}^\sigma(k) + \hat{\varepsilon}^\sigma(k) \frac{i(\hat{p} - \hat{l}) - m}{(p-l)^2 + m^2} \hat{\varepsilon}^\tau(l) \right) u_r(p) \quad (80)$$

Here $\hat{\varepsilon}^\sigma(k) = \varepsilon_\mu^\sigma(k) \gamma_\mu$ is Lorentz invariable, but $\hat{p} = p_\mu r_\mu$, $\hat{k} = k_\mu r_\mu$ and $\hat{l} = l_\mu r_\mu$ which come from the propagation function of electron have no symmetries of Lorentz transformation. We have

$$\begin{aligned} \hat{p} &\rightarrow a_{\nu\mu} a_{\mu\lambda} p'_\mu \gamma_\lambda = \frac{1+V^2}{1-V^2} p'_1 \gamma_1 + \frac{2iV}{1-V^2} p'_1 \gamma_4 \\ &- \frac{2iV}{1-V^2} p'_4 \gamma_1 + \frac{1+V^2}{1-V^2} p'_4 \gamma_4 + p'_2 \gamma_2 + p'_3 \gamma_3 \neq \hat{p}' \end{aligned} \quad (81)$$

And so do for \hat{k} and \hat{l} . So we have $M'_{fi} \neq M_{fi}$. The second process of Compton scattering has non-relativity. For the second order process containing the propagation line of bosons, M_{fi} is symmetric under Lorentz transformation. But in the more high order processes of Compton scattering, the propagation line of fermion will appear. The processes still have no symmetry of Lorentz transformation.

6.3 The Lorentz transformations of probability transformations relative to photon's polarization.

For the interaction processes related to photons, the transition probabilities are related to the summation of photon's polarization. Because the polarizations of photons are not detected in experiments, we should take the average over the polarization of photon in the initial states and take the summation over the polarization of photon in the final states. Because photons are transversely polarized, both longitudinal and time photons are considered untrue. They have no contribution for the transition probability. The summation formula of polarization is **【21】**:

$$\begin{aligned} \sum_{\sigma=1}^2 \sum_{\tau=1}^2 (\varepsilon^\sigma(k) \varepsilon^\tau(l))^2 &= \sum_{\sigma=1}^2 \sum_{\tau=1}^2 (\varepsilon_\mu^\sigma(k) \varepsilon_\mu^\tau(l)) (\varepsilon_\nu^\sigma(k) \varepsilon_\nu^\tau(l)) \\ &= \sum_{\sigma=1}^2 (\varepsilon_\mu^\sigma(k) \varepsilon_\nu^\tau(k)) \sum_{\tau=1}^2 (\varepsilon_\mu^\sigma(l) \varepsilon_\nu^\tau(l)) \end{aligned} \quad (82)$$

It is easy to know without concrete calculation that the formula has no Lorentz invariability due to the fact that σ and τ only take the values 1 and 2. In fact, according to the current method, by considering (73) and (74), we get

$$\sum_{\sigma=1}^2 \varepsilon_\mu^\sigma(k) \varepsilon_\nu^\tau(k) = \sum_{\sigma=1}^4 \varepsilon_\mu^\sigma(k) \varepsilon_\nu^\tau(k) - \frac{1}{\omega_k^2} (k_\mu k_\nu - i\omega_k (k_\mu \delta_{\nu 4} + k_\nu \delta_{\mu 4}))$$

$$= \delta_{\mu\nu} - \frac{1}{\omega_k^2} (k_\mu k_\nu - i\omega_k (k_\mu \delta_{\nu 4} + k_\nu \delta_{\mu 4})) \quad (83)$$

For example, for the process of Compton scattering, the result is **[2]**:

$$\begin{aligned} \sum_{\sigma=1}^2 (\varepsilon_\mu^\sigma(k) \varepsilon_\nu^\tau(k)) \sum_{\tau=1}^2 (\varepsilon_\mu^\sigma(l) \varepsilon_\nu^\tau(l)) &= \left\{ \delta_{\mu\nu} - \frac{1}{\omega_k^2} (k_\mu k_\nu - i\omega_k (k_\mu \delta_{\nu 4} + k_\nu \delta_{\mu 4})) \right\} \\ &\times \left\{ \delta_{\mu\nu} - \frac{1}{\omega_l^2} (l_\mu l_\nu - i\omega_l (l_\mu \delta_{\nu 4} + l_\nu \delta_{\mu 4})) \right\} = 1 + \cos^2 \theta \end{aligned} \quad (84)$$

In the formula, (73) is used. Because (73) has no Lorentz invariability, (84) has no invariability too. That is to say, all micro-interaction processes related photons have no Lorentz invariability.

7 . The Lorentz transformations of interaction processes between bound state's particles

7.1 The motion equations of bound particles

According to present classification, the theories of micro-physics are divided into relativity quantum theory of field and non-relativity approximated quantum mechanics. Quantum theory of field describes unstable particles with high moving speed, mainly used in the interaction processes between elementary particles which are free at their initial and final states. The interaction Hamiltonians can be constructed by using the wave functions of free particles. As shown in (14) and (15), the wave function of free particle is invariable under Lorentz transformation. The amplitudes of transition probabilities can also be represented by the products of four-dimensional energy momentums of free particles just $p \cdot q$ which are the invariable quantity of Lorentz transformation.

However, fundamental particles physics is only a branch of physics. These particles are created in laboratory and then decay immediately. So quantum theory of field can only dealt a small part of physics which has no closed relation with real world. What closely connected to practical world is the interaction between bound particles, for example, atoms emitting photos, superconductors and condensed matter and so on. In these problems, the interaction Hamiltonians can not be constructed by free particle's wave functions. For non-free particles, we have $p^2 \neq -m^2$. The product $p \cdot q$ is not the invariable quantity of the Lorentz transformation. The method of quantum theory of field may not be effective in general.

Non-relativity quantum mechanics describe stable particles with low moving speed. However, the principle of relativity has no approximation. It is either tenable or not tenable. If it is tenable, it should hold for the particles moving in low speeds. In fact, the principle of relativity is considered to be tenable in classical Newtonian mechanics. The formulas of the Newtonian mechanics are considered unchanged under the Galileo's transformation. The motion equations of non-relativity quantum mechanics can not keep unchanged under the Lorentz transformation. Can it be invariable under the Galileo's transformation? The answer is neglected. For example, the Schrodinger's equation is

$$i\hbar \frac{\partial}{\partial t} \psi(\bar{x}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\bar{x}, t) + V(\bar{x}, t) \psi(\bar{x}, t) = \hat{H}(\bar{x}, t) \psi(\bar{x}, t) \quad (85)$$

It is not invariable no matter for the Lorentz transformation or the Galilee's transformation. In fact, no

matter in quantum mechanics or quantum theory of field, even the most foundational normalization formula of probability wave in the three-dimensional coordinates space has no the invariability of Lorentz transformation.

$$1 = \iiint \psi^*(\bar{x}, t) \psi(\bar{x}, t) dx dy dz \rightarrow \frac{1}{\sqrt{1-V^2}} \iiint \psi^*(\bar{x}', V, t') \psi(\bar{x}', V, t') (dx' + V dt') dy' dz' \neq \iiint \psi^*(\bar{x}', t') \psi(\bar{x}', t') dx' dy' dz' \quad (86)$$

So generally speaking, quantum theories of common micro-physics have no relativity. So-called the non-relativity of bound particle's motion equations is not due to the approximate description methods we used. The essence is that they have no relativity at all!

We take electron's scattering in external field, fine structure of hydrogen atomic energy level and light's emission and absorption in bound atoms as examples to prove this conclusion below. The Dirac equation of quantum mechanics is based on special relativity. However, it becomes non-relativity when it is used to describe the energy levels of bound hydrogen atoms.

7.2 The Lorentz transformation of electron's scattering in external field

Suppose that the external field is static electric field with form $\hat{A}(x) = i\gamma_4 V(x)$. The interaction Hamiltonian is

$$\mathcal{H}(x) = -ie \bar{\psi}(x) \hat{A}(x) \psi(x) = eV(x) \bar{\psi}(x) \gamma_4 \psi(x) \quad (87)$$

(87) is not the Lorentz invariable quantity. Based on it, it is proved in quantum theory of field that the effective scattering section is **【1】**

$$d\sigma_{fi} = \frac{m^2 \alpha^2 Z^2}{4|\bar{p}|^4 \sin^2 \theta/2} \sum |U_{fi}|^2 d\Omega(\theta, \varphi) \quad (88)$$

The transition probability amplitude is

$$\sum |U_{fi}|^2 = \frac{1}{2} \text{Tr} \gamma_4 A_+(p_1) \gamma_4 A_+(p_2) = \frac{m^2 + E_1 E_2 + \bar{p}_1 \cdot \bar{p}_2}{2m} \quad (89)$$

Here E_1 and \bar{p}_1 are the energy and momentum of incident particle and E_2 and \bar{p}_2 are that of outgoing particle. We have

$$E_1 E_2 + \bar{p}_1 \cdot \bar{p}_2 = \frac{m^2 (1 + u_{1x} \cdot u_{2x} + u_{1y} \cdot u_{2y} + u_{1z} \cdot u_{2z})}{\sqrt{1-u_1^2} \sqrt{1-u_2^2}} \quad (90)$$

By transforming (88) to moving reference frame, we have

$$E'_1 E'_2 + \bar{p}'_1 \cdot \bar{p}'_2 = \frac{m^2 \left[(1+V^2)(1+u'_{1x} u'_{2x}) + 2V(u'_{1x} + u'_{2x}) \right]}{(1-V^2) \sqrt{1-u_1'^2} \sqrt{1-u_2'^2}} + \frac{m^2 (u'_{y1} u'_{y2} + u'_{z1} u'_{z2})}{\sqrt{1-u_1'^2} \sqrt{1-u_2'^2}} \quad (91)$$

So (91) is not the Lorentz invariable quantity. In addition, phase space factor is not unchanged too, (88) has no relativity.

7.3 The Lorentz transformation of fine structure of hydrogen atomic energy level

It is obvious that the energy levels of hydrogen atom can not be calculated by quantum theory of field,

though it can be used to calculate the high order revision (Lams shift). By considering relativity quantum mechanics, the motion equation of an electron in hydrogen atom is

$$E\psi = (\alpha \cdot \bar{p} + \beta m + e\phi) \quad (91)$$

Here $\phi(r)$ is electron's potential function. Dirac used (91) to do calculation and obtained the formula of fine structure of hydrogen atomic energy level **【4】**

$$E_n = mc^2 \left[1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{2n^4} \left(\frac{n}{|K|} - \frac{3}{4} \right) + \dots \right] \quad (92)$$

However, $\phi(r) = e/r$ is not the Lorentz invariability quantity with

$$\phi(x, y, z) = \frac{e}{\sqrt{x^2 + y^2 + z^2}} \rightarrow \frac{e}{\sqrt{\frac{(x' + Vt')^2}{1 - V^2} + y'^2 + z'^2}} = \phi'(x', y', z', t', V) \quad (93)$$

After the transformation (for simplification, only considering direct Lorentz transformation, without considering the relativity transformation), the potential is related to speed and time. It is impossible for us to obtain the energy levels if we do calculation in K' reference frame by using $\phi'(x', y', z', t', V)$ as potential. So the fine structure of hydrogen atomic energy level has no relativity.

7.4 The Lorentz transformation of light's emission and absorption in bound atoms

In the radiant process of common material, interaction between electromagnetic wave and bound electron in atom is involved. Suppose that electromagnetic wave transits along z axis, according to non-relativity quantum mechanics, the interaction Hamiltonian is **【5】**:

$$\hat{H} = -ie \frac{\hbar \alpha}{2m} \left[\exp[i(k \cdot z - \omega t)] \frac{\partial}{\partial x} + \exp[-i(k \cdot z - \omega t)] \frac{\partial}{\partial x} \right] \quad (94)$$

Here free plane wave $\exp[i(k \cdot z - \omega t)]$ is Lorentz invariable, but operator $\partial/\partial x$ is not. According to the (32), its transformation is

$$\frac{\partial}{\partial x} \rightarrow \frac{1}{\sqrt{1 - V^2}} \frac{\partial}{x'} - \frac{V}{\sqrt{1 - V^2}} \frac{\partial}{\partial t'} \quad (95)$$

The operator is acted on the wave function of bound electron and the bound wave function is not Lorentz invariable. The result has no symmetry of Lorentz transformation. Similarly, (94) is not invariable under the Galileo's coordinate transformations. No matter from what angle, the processes of light's emission and absorption have no relativity too.

In fact, for most practical problems of micro-physics, we can not use the method of relativity quantum theory of field to deal with. We can only calculate them by non-relativity quantum mechanics. Under the conditions of low speeds, these methods are very effective. We can not say they are imprecise. More important is that these problems have no the symmetries of Lorentz transformation in essence, we can not impose relativity on them.

8. The Lorentz transformations of high order perturbations

normalization processes in quantum theory of field

8. 1 The Lorentz transformation of Lamb shift

The high order perturbation processes of quantum theory of fields contain infinite and need to be renormalized. The normalization may introduce the symmetry violation of Lorentz transformation. At first, we take the Lamb shift of hydrogen atom's energy levels as examples. By the normalization calculation, the Lamb shift is **【6】**

$$\Delta E_n = \frac{4\alpha^2}{3m^2} \left(\ln \frac{m_0}{2\varepsilon_0} - \frac{3}{8} - \frac{1}{5} + \frac{5}{6} \right) \cdot_n \langle |\nabla^2 V(\bar{x})| \rangle_n + \frac{i\alpha^2}{4\pi m} \cdot_n \langle |\beta \bar{\alpha} \cdot \nabla(1/r)| \rangle_n \quad (96)$$

Here $|\rangle_n$ is the wave function of bound electron in hydrogen atom. For the ground state's electron, we have

$$|\rangle_1 = \frac{1}{\sqrt{\pi\alpha^{3/2}}} e^{-\frac{r}{\alpha}} = \frac{1}{\sqrt{\pi\alpha^{3/2}}} \exp\left(-\frac{\sqrt{x^2 + y^2 + z^2}}{\alpha}\right) \quad (97)$$

By transforming it to moving reference frame, we get

$$|\rangle_1 \rightarrow \frac{1}{\sqrt{\pi\alpha^{3/2}}} \exp\left(-\frac{1}{\alpha} \sqrt{\frac{(x' + Vt')^2}{1-V^2} + y'^2 + z'^2}\right) \quad (98)$$

New function depends on time and relative speed without Lorentz invariability. Operators $\nabla^2 V(\bar{x})$ and $\bar{\alpha} \cdot \nabla(1/r)$ are not invariable too. Both average values $\cdot_n \langle |\nabla^2 V(\bar{x})| \rangle_n$ and $\cdot_n \langle |\beta \bar{\alpha} \cdot \nabla(1/r)| \rangle_n$ are not symmetrical under the Lorentz transformation. So the Lamb shift has no relativity.

8. 2 The Lorentz symmetry violations caused by integral transformations

In the normalization processes of high order perturbations, following integrals are involved

$$\int_{-\infty}^{\infty} d^4 k \frac{f(k)}{(k-l)^2 + b^2} \quad \text{and} \quad \int_{-\infty}^{\infty} d^4 k \frac{k_\mu - l_\mu + c}{(k-l)^2 + b^2} \quad (99)$$

Here $k = (\bar{k}, ik_0)$, $k^2 = \bar{k}^2 - k_0^2$ and $d^4 k = dk_1 dk_2 dk_3 dk_0$. As we known that k_y and k_z are Lorentz invariable, but k_x and k_0 are not. By using the Jacobi's formula, we have

$$\begin{aligned} dk_x dk_0 &= \begin{vmatrix} \partial k_x / \partial k'_x & \partial k_0 / \partial k'_0 \\ \partial k_0 / \partial k'_x & \partial k_x / \partial k'_0 \end{vmatrix} dk'_x dk'_0 \\ &= \begin{vmatrix} 1/\sqrt{1-V^2} & V/\sqrt{1-V^2} \\ V/\sqrt{1-V^2} & 1/\sqrt{1-V^2} \end{vmatrix} dk'_x dk'_0 = dk'_x dk'_0 \end{aligned} \quad (100)$$

So $d^4 k$ is still the invariable quantity of Lorentz transformation. Because $k^2 = \bar{k}^2 - k_0^2 = -m^2$ is also Lorentz invariable, (99) is unchanged under the Lorentz transformation.

However, the direct calculation of (99) is difficult. In order to complete the integrals, we need to move the original point of coordinate, then introduce transformation $k_0 \rightarrow ik_0$ and obtain $dk_1 dk_2 dk_3 dk_0 \rightarrow idk_1 dk_2 dk_3 dk_0$ as well as $k^2 \rightarrow \bar{k}^2 + k_0^2 = K^2$. By introducing four-dimensional spherical coordinates with [4]

$$k_0 = K \cos \Phi \quad k_1 = K \sin \Phi \sin \theta \cos \varphi$$

$$k_2 = K \sin \Phi \sin \theta \sin \varphi \quad k_3 = K \sin \Phi \cos \varphi \quad (101)$$

we get

$$d^4k = K^3 \sin^2 \Phi \sin \theta dK d\theta d\varphi \quad (102)$$

In this way, for same simple situations, (99) can be integrated. For example

$$\int_{-\infty}^{\infty} \frac{d^4k}{k^2 + b^2} = \frac{i\pi^2}{2b^2} \quad J_n = \int_{-\infty}^{\infty} \frac{d^4k}{(k^2 + b^2)^3} = \frac{i\pi^2}{(n-1)(n-2)b^{2(n-2)}} \quad (103)$$

Let K'_L is the Lorentz transformation of K , we have

$$\begin{aligned} k^2 = K^2 \rightarrow K_L'^2 &= \frac{m^2[(u'_x + V)^2 + (1 + u'_x V)^2]}{(1 - u'^2)(1 - V^2)} + k_y'^2 + k_z'^2 \\ &= \frac{1+V^2}{1-V^2} k_x'^2 + \frac{1+V^2}{1-V^2} k_0'^2 + \frac{4V}{1-V^2} k'_x k'_0 + k_y'^2 + k_z'^2 \neq k_x'^2 + k_y'^2 + k_z'^2 + k_0'^2 = K'^2 \end{aligned} \quad (104)$$

So k^2 is not the invariable quantity of Lorentz transformation and (99) and (103) are not invariable again. Because (99) and (103) appears in the common normalization processes of high order perturbations commonly, the calculation methods of normalizations violate the Lorentz invariability generally. Let's take several examples below.

8.3 The Lorentz transformation of mass normalization

The probability amplitude of mass normalization is 【6】

$${}_f \langle |S^{(2)}| \rangle_i = i(2\pi)^4 \delta^4(p - p_2) \bar{u}_2(p_2) A u_1(p_1) \quad (105)$$

$$A = \frac{e^2 m}{(2\pi)^4} \left(\frac{\pi^2}{2} + 2i \int d^4k \int_0^1 \frac{(1+x)dx}{(k^2 + m^2 x^2)^2} \right) \quad (106)$$

$$\int d^4k \int_0^1 dx \frac{1+x}{(k^2 + m^2 x^2)^2} = \frac{3}{2} \int \frac{d^4k}{(k^2 + m^2)^2} + 2m^2 \int d^4k \int_0^1 \frac{x^2(1+x)dx}{(k^2 + m^2 x^2)^3} \quad (107)$$

By introducing $k_0 \rightarrow ik_0$ and considering $k^2 \rightarrow K^2$, we have

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{d^4k}{(k^2 + m^2)^2} &\rightarrow i\pi^2 \int_0^{\infty} \frac{K^2 dK^2}{(K^2 + m^2)^2} \\ &= i\pi^2 \lim_{K \rightarrow \infty} \left(\frac{K^2 + m^2}{m^2} + \frac{m^2}{K^2 + m^2} - 1 \right) = i\pi^2 D \end{aligned} \quad (108)$$

$$2m^2 \int d^4k \int_0^1 \frac{x^2(1+x)dx}{(k^2 + m^2 x^2)^2} \rightarrow 2m^2 \int d^4K \int_0^1 \frac{x^2(1+x)dx}{(K^2 + m^2 x^2)^3} = \frac{5}{2} i\pi^2 \quad (109)$$

D in (108) is linearly infinite, but (109) is finite. Because $d^4k = id^4K \rightarrow id^4K'$ is Lorentz invariable, but $K^2 \rightarrow K_L'^2$ is not according to (104), we have

$$\int_{-\infty}^{\infty} \frac{d^4 k}{(k^2 + m^2)^2} \rightarrow i\pi^2 \int_0^{\infty} \frac{K'^2 dK'^2}{(K_L'^2 + m^2)^2} \neq i\pi^2 D \quad (110)$$

$$2m^2 \int d^4 k \int_0^1 \frac{x^2(1+x)dx}{(k^2 + m^2 x^2)^2} \rightarrow 2m^2 \int d^4 K' \int_0^1 \frac{x^2(1+x)dx}{(K_L'^2 + m^2 x^2)^3} \neq \frac{5}{2} i\pi^2 \quad (111)$$

The probability amplitude of mass normalization has no invariability of Lorentz transformation.

8.4 The Lorentz transformation of vacuum polarization

The probability amplitude of vacuum polarization is **【6】**

$$\begin{aligned} {}_f \langle |S^{(2)} + S^{(4)}| \rangle_i &= e^2 (2\pi)^8 \int d^4 k \bar{u}_3(p_3) \gamma_\mu u_1(p_1) D_{f,\mu\nu}^{(2)}(k) \bar{u}_4(p_4) \gamma_\nu u_2(p_2) \\ &\quad \times \delta^4(p_1 - p_3 - k) \delta^4(k + p_2 - p_4) \end{aligned} \quad (112)$$

$$D_{f,\mu\nu}^{(2)}(k) = \delta_{\mu\nu} D_f(k) + D_f(k) \Pi_{\mu\nu}^{(2)}(k) D_f(k) \quad (113)$$

$$\begin{aligned} \Pi_{\mu\mu}^{(2)}(k) &= 16e^2 \int d^4 p \int_0^1 dy \frac{(1-y)(p^2 + 2m^2 - (p \cdot k))(2(p \cdot k) - k^2)}{((p - ky)^2 + m^2 + k^2 y(1-y))^3} \\ &= 3(CD_f^{-1}(k) + \Pi_f^{(2)}(k^2)D_f^{-1}(k)) \end{aligned} \quad (114)$$

Here C is infinite, we have

$$C = -\frac{\alpha}{3\pi} \left(D + \frac{5}{6} \right) \quad (115)$$

By taking several simplifications, including

$$(p \cdot k)^2 = p_\mu p_\nu k_\mu k_\nu \rightarrow \frac{1}{4} p^2 k^2 \quad (116)$$

we obtain

$$\begin{aligned} \Pi_f^{(2)}(k^2) &= \frac{\alpha}{24\pi^7} 16e^2 \int d^4 p \int_0^1 dy y(1-y)^2(2y-1) \\ &\quad \times \left[\frac{2}{(p^2 + m^2 + k^2 y(1-y))^3} + \int_0^1 \frac{dz 3(p^2 + 4m^2)}{(p^2 + m^2 + k^2 yz(1-y))^4} \right] \end{aligned} \quad (117)$$

Let $p_0 \rightarrow ip_0$ and $p^2 \rightarrow K^2$, the integral of (117) is

$$\Pi_f^{(2)}(k^2) = \frac{i2\alpha}{3(2\pi)^5} \int_0^1 dy y \frac{(2y-1)(2y-3)}{m^2 + k^2 y(1-y)} \quad (118)$$

Similarly, because $d^4 p \rightarrow id^4 K$ is invariable but $p^2 \rightarrow K_L'^2 \neq K'^2$ is not under the Lorentz transformation. So (117) is not invariable. We can not obtain (118) from (117) after Lorentz transformation. Besides, (114) contains $p \cdot k$. If we take $p_0 \rightarrow ip_0$ in (114), correspondingly, we should let $k_0 \rightarrow ik_0$ in (117), so that k^2 in (117) and (118) is not Lorentz invariable again. But this point was neglected in the

current theory of normalization.

8.5 The Lorentz transformation of third order vertex angle process

The amplitude of third order vertex angle process is 【6】

$$f \langle |S^{(3)}| \rangle_i = -e(2\pi)^4 \delta^4(p_2 - p_1 - k_1) \bar{u}_2(p_2) A_\mu^{(2)}(p_1, p_2) u_1(p_1) a_\nu(k_1) \quad (119)$$

Set $A_\mu^{(2)}(p_1, p_2) = L\gamma_\mu$, we get

$$\begin{aligned} L &= \frac{i\alpha}{2\pi^3} \int d^4k \int_0^1 dx \cdot x \left[\frac{m^2(4-4x-x^2)}{(k^2+m^2x^2)^3} - \frac{1}{(k^2+m^2x^2)^2} \right] \\ &= \frac{\alpha D}{4\pi} + \frac{i\alpha}{2\pi^3} \int d^4k \int_0^1 dx \frac{xm^2(4-4x-x^2)}{(k^2+m^2x^2)^3} = \frac{\alpha}{4\pi} \left(D - 4 \int_0^1 \frac{dx}{x} + \frac{11}{2} \right) \end{aligned} \quad (120)$$

Similarly, $d^4k \rightarrow id^4K$ is Lorentz invariable but $k^2 \rightarrow K_L'^2 \neq K'^2$ is not. We have

$$L' \neq \frac{\alpha}{4\pi} \left(D - 4 \int_0^1 \frac{dx}{x} + \frac{11}{2} \right) \quad (121)$$

The normalization of the third order vertex angle process is not invariable under Lorentz transformation.

It notes that this kind of symmetry violation is caused by the calculation method of integral transformation $p_0 \rightarrow ip_0$. If we do not introduce this transformation, there is no symmetry violation. We should to ask whether or not the symmetry violation is essential? The problem is that if we do not let $p_0 \rightarrow ip_0$, the integral can not be completed so that concrete results can not be obtained to compare with experiments. We should consider the integral transformation as a part of normalization calculation. Because the results of normalizations are consistent with experiments, in this meaning, we may say that the calculation method which leads to Lorentz symmetry violation has the practical meaning in physics.

In summary, three basic normalizations processes of electromagnetic interaction violate the symmetry of Lorentz transformation. The conclusion is also suitable for other interaction theories.

9. Discussion

By the analysis above, we see that the principle of relativity can not hold in the interaction theories of micro-particles at all! Einstein's special relativity is mainly used in microscopic high speed processes. If relativity principle does not hold in micro-processes, it does not exist in nature.

However, why physicists did not find this problem up to now? This question is worthy of our thought. When Einstein put forward special relativity in 1905, science community has not yet reached common understanding about whether or not atoms exist, not to mention quantum mechanics and elementary particle physics. It was the matter after relativity was accepted widely when it was applied in elementary particle physics. Physicists believed relativity was correct, so that Lorentz symmetry violation in micro-physics was neglected consciously or unconsciously. Speaking in other words, the problems of Lorentz symmetry violations in micro-particle physics were handled vaguely. However, as long as we get to the bottom of matter, the problems still emerge from the water.

In special relativity, the invariability principle of light's speed and the principle of relativity are independent each other. But in some situations, they are connected. The Lorentz formula is deduced based on the invariability principle of light's speed. According to the principle of relativity, the Lorentz formula

has relative significance only. Meanwhile, the principle of relativity declares that the forms of physical motion equations do not change with reference frames. To reach this aim, physical quantities should be transformed in covariant forms. As shown in (41) and (42), covariance is for general physical quantities and Lorentz transformation is for space-time coordinate. Both are different concepts, but we often do not distinguish them.

Besides, in order to keep the motion equation of spinor field unchanged, we should introduce spinor transformation. However, as shown in (36), spinor transformation is not covariant. That is to say, in order to make the motion equation of quantum mechanics unchanged, covariance is not enough. The problem is very complex actually. It is proved in this paper that even though spinor transformation is considered, the interaction theories of micro-particles have yet no relativity.

On the other hand, the invariability principle of light's speed has obtained a lot of verifications, especially in the experiments of high energy accelerators. For example, the experiment to measure light's speed in the process that high energy proton decays into meson and photon in CERN in 1964. However, the principle of relativity is only a kind of belief without really strict verification. Because common experiments only involve low speed motions, similar to Galileo, modern physicists make their judgments according to common experiments. They believe that the principle of relativity is alright.

So the real situations may be that the effectiveness of the invariability principle of light's speed covers the ineffectiveness of the principle of relativity. In fact, up to now days, physicists have never made practical measurements in the moving reference frame with high enough speed to verify the principle of relativity. The Michelson interference experiment seems to prove that the absolute motion of the earth can not be measured. However, in relativity, we use the invariability of light's speed to explain the Michelson interference experiment. In this meaning, the Michelson interference experiment only verifies the invariability principle of light's speed, without verifying the principle of relativity.

Ironically, many pioneers of relativity including Michelson, Lorentz, Poincare and March and so on did not accept Einstein's relativity. What they opposed was the principle of relativity, rather than the invariability principle of light's speed. The reason was that the principle of relativity leads to various logic paradoxes. In fact, it is the problem of experiment whether or not the invariability principle of light's speed can hold. If there is no the principle of relativity, we can still reach the Lorentz transformation based on the invariability principle of light's speed. In this case, the Lorentz transformation becomes absolute, just as Lorentz himself considered it. All formulas of special relativity are still effective, but become absolute ones. The effects of special relativity also become absolute ones.

However, it is mainly a logic problem whether or not the principle of relativity is tenable. So it is easy to cause arguments. Since Einstein established special relativity one hundred years ago, criticism has never stopped. The arguments are so violent and last so long time that it is unwonted in the history of science. The arguments are concentrated on the space-time paradox problems just as length paradox and time paradox which are often specious. Correctness and mistakes are mixed together so that it is difficult to obtain correct judgment. Because these problems do not belong to the main stream of physical research at present time, physicists do not pay attention to them again.

A dramatic change appeared in 1960's when CMBR was founded. The isotropy of CMBR provided a choice of absolutely static reference frame for physics. The principle of relativity is not a problem of logic again. It became a problem which can be verified through experiments and observation of astronomy. In fact, the Sagnac effect found in 1912 had indicated that the relativity of motion was impossible. The time comparison experiments of microwave communications through satellite between Xian and Tokay achieved

in 2001 also revealed the same conclusion 【7】 . But what can really provide an absolutely static reference frame for physics is just the isotropy of CMBR. Because it contradicts the relativity principle of motion directly, physics is in an awkward position at present.

On the other hand, all arguments about relativity principle such as space-time paradox, Signac effect, or the isotropy of CMBR are macro-phenomena. But macrocosm is composed of micro-particles. If the physical laws of micro-particles have relativity, physicists have reason to believe the principle of relativity. But if the physical laws of micro-particles have no invariability under the Lorentz transformations, it means that micro-physics has no relativity, physicists have no reason to stick to the relativity principle of motion again.

The proofs in this paper are clear and certain without any speciousness. So there is only a way for physics. That is to give up the principle of relativity but reserve the invariability principle of light's speed. In this way, all formulas of special relativity can be reserved but we need to explain them in absolute forms. The author will discuss this problem later.

About three hundred years ago, Newton established classical mechanics. Newton thought that absolute space existed, but he did not know where it was. Modern cosmology found absolute reference frame for physics. Particle physics will also provide its judgment for absolute motion. Both macro-physics and microphysics reach united conclusion again.

Appendix: The mistakes in the proof of the Lorentz transformation invariability of spinor field's in quantum theory of field

In the textbook “Introduce to Quantum Theory of Field” by Luo Changxun, p202, the definition of (15) and (27.a) are

$$S_{\alpha\beta}^{(\pm)}(x_1 - x_2) = (\gamma_\mu \partial_\mu - m)_{\alpha\beta} \Delta^{(\pm)}(x_1 - x_2) \quad (15)$$

$$\{\psi_\alpha^{(-)}(x_1), \bar{\psi}_\beta^{(+)}(x_2)\} = -iS_{\alpha\beta}^{(-)}(x_1 - x_2) \quad (16a)$$

Let
$$\psi(x) \rightarrow \psi'(x') = \Lambda \psi(x) \quad (21a)$$

We have
$$\psi'_\sigma(x'_1) = \Lambda_{\sigma\alpha} \psi_\alpha(x_1) \quad \bar{\psi}'_\rho(x'_2) = \bar{\psi}_\beta(x_2) \Lambda_{\beta\rho}^{-1} \quad (21)$$

Substituting in (16a) , it is obtained

$$\begin{aligned} \{\psi'^{(-)}_\sigma(x'_1), \bar{\psi}'^{(+)}_\rho(x'_2)\} &= \Lambda_{\sigma\alpha} \{\psi^{(-)}_\alpha(x_1), \bar{\psi}^{(+)}_\beta(x_2)\} \Lambda_{\beta\rho}^{-1} \\ &= \Lambda_{\sigma\alpha} [(i\gamma_\mu)_{\alpha\beta} \partial_\mu - im\delta_{\alpha\beta}] \Lambda_{\beta\rho}^{-1} \Delta^{(+)}(x_1 - x_2) \\ &= \Lambda_{\sigma\alpha} [(i\gamma_\mu)_{\alpha\beta} \partial_\mu - im\delta_{\alpha\beta}] \Lambda_{\beta\rho}^{-1} \Delta^{(+)}(x'_1 - x'_2) \end{aligned} \quad (22)$$

Considering the Lorentz transformation invariability of the commutation relation of scalar field in (22)

$$\Delta^{(+)}(x'_1 - x'_2) = \Delta^{(+)}(x_1 - x_2) = \frac{1}{(2\pi)^4} \int \frac{d^3 p}{2E} e^{ip \cdot (x_1 - x_2)} \quad (23)$$

Using the formula (32) and (35) in this paper, it can be obtained

$$\Lambda_{\sigma\alpha}(\gamma_\mu)_{\alpha\beta} \Lambda_{\beta\rho}^{-1} \partial_\mu = (a_{\mu\nu} \gamma_\nu \partial_\mu)_{\sigma\rho} = (\gamma_\nu \partial'_\nu)_{\sigma\rho} \quad (24)$$

So it is considered to have the invariability of Lorentz transformation

$$\{\psi'^{(-)}_\sigma(x'_1), \bar{\psi}'^{(+)}_\rho(x'_2)\} = -i(\gamma_\mu \partial'_\mu + m)_{\sigma\rho} \Delta^{(+)}(x'_1 - x'_2) = -iS^{(+)}_{\sigma\rho}(x'_1 - x'_2) \quad (25)$$

There are four mistakes in the proof above

1. (21a) is wrong. According the definition of spinor transformation $\psi'(x') = \Lambda \psi(x)$, correct one is

$$\psi(x) = \Lambda^{-1} \psi'(x') \quad \bar{\psi}(x) = \psi'(x') \Lambda \quad (A1)$$

Substituting in (16a), the formula (62) in this paper is obtained.

$$\begin{aligned} \{\psi^{(-)}_\alpha(x_1), \bar{\psi}^{(+)}_\beta(x_2)\} &= \Lambda_{\alpha\sigma}^{-1} \{\psi'^{(-)}_\sigma(x'_1), \bar{\psi}'^{(+)}_\rho(x'_2)\} A_{\rho\beta} \\ &= \Lambda_{\alpha\sigma}^{-1} [(i\gamma_\mu)_{\sigma\rho} a_{\nu\mu} \partial'_\nu - im \delta_{\sigma\rho}] A_{\rho\beta} \Delta_L^{(+)}(x'_1 - x'_2) \end{aligned} \quad (A2)$$

Therefore, (22) is wrong.

2. As proved in this paper, the phase space factor $d^3 p / E$ is not an invariability of Lorentz, so (23) does not hold.

3. According to Luo Zhangxun's book, the formula (55b) in p. 136, the first equal sign in (24) is wrong. Correct one is

$$\Lambda_{\sigma\alpha}(\gamma_\mu)_{\alpha\beta} \Lambda_{\beta\rho}^{-1} \partial_\mu = (a_{\nu\mu} \gamma_\nu \partial_\mu)_{\sigma\rho} \quad (A3)$$

4. According to Luo Zhangxun's book, the formula (52) in p. 136, the second equal sign in (24) is wrong. Correct one is

$$a_{\mu\nu} \gamma_\nu \partial_\mu = a_{\mu\nu} \gamma_\nu a_{\lambda\mu} \partial'_\lambda \neq \delta_{\nu\lambda} \gamma_\nu \partial'_\lambda = \gamma_\nu \partial'_\nu \quad (A4)$$

In fact, if (21a) hold, the mistake in (A3) and (A4) offsets each other. But because $d^3 p / E$ is not an invariability, (23) does not hold, so that (25) is still untenable.

It is noted in the Luo's textbook that the transformations are carried out three times. Form no primed propagation functions to primed propagation function, then to no primed propagation function again. Why?

If the this method is used to the transformation of interaction Hamiltonian, according to (21a)、(35) and (44), we get

$$\begin{aligned}
A_\mu(x)\bar{\psi}(x)\gamma_\mu\psi(x) &\rightarrow A'_\mu(x')\bar{\psi}'(x')\gamma_\mu\psi'(x') = a_{\mu\nu}A_\nu(x)\bar{\psi}(x)\Lambda^{-1}\gamma_\mu\Lambda\psi(x) \\
&= a_{\mu\nu}a_{\mu\lambda}A_\nu(x)\bar{\psi}(x)\gamma_\lambda\psi(x) = \delta_{\nu\lambda}A_\nu(x)\bar{\psi}(x)\gamma_\lambda\psi(x) = A_\nu(x)\bar{\psi}(x)\gamma_\nu\psi(x) \quad (\text{A5})
\end{aligned}$$

(A5) becomes identical relation. So the transformation (21a) is meaningless.

In fact, if discussing the transformation from primed propagation to no primed function, we should start from (A6) with

$$\{\psi'_\alpha(x'_1), \bar{\psi}'_\beta(x'_2)\} = (\gamma_\mu\partial'_\mu - m)_{\alpha\beta}\Delta'^{(\pm)}(x'_1 - x'_2) = -iS'^{(-)}_{\alpha\beta}(x'_1 - x'_2) \quad (\text{A6})$$

It notes that the sign ∂'_μ in (A6) is primed. Then take the transformation

$$\psi'(x') = \Lambda\psi(x) \quad (\text{A7})$$

$$\psi'_\sigma(x'_1) = \Lambda_{\sigma\alpha}\psi_\alpha(x_1) \quad \bar{\psi}'_\rho(x'_2) = \bar{\psi}_\beta(x_2)\Lambda^{-1}_{\beta\rho} \quad (\text{A8})$$

It can be proved that (A6) has no invariability of Lorentz transformation.

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